Fermi National Accelerator Laboratory

FN-272
1100.050

CASIM

(First Edition)

PROGRAM TO SIMULATE TRANSPORT OF HADRONIC CASCADES IN BULK MATTER

A. Van Ginneken

January 1975

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

CONTENTS

I. INTRODUCTION

CASIM is a Monte Carlo (MC) program which studies the average development of internuclear cascades when high energy particles are incident on large targets (shields) of arbitrary geometry and composition. The program aims at problems for incident particles in the range 20 - 1000 GeV/c. It does not study the transport of low energy particles ($\lesssim$ 0.3 GeV/c). In contrast to similar programs developed elsewhere, the program uses directly inclusive distributions i.e., particle yields as a function of angle and momentum (or other equivalent variables) from inelastic particle-nucleus interactions.[1,2] The use of weighting techniques avoids difficulties of random selection encountered in sampling complicated distributions (see below) and allows the user to introduce bias in the sampling.[3]

From the particle production model (in the form of a set of inclusive distributions) and a few other inputs (range-momentum relationship, inelastic interaction lengths, etc.) plus a geometry subroutine (generally, supplied by the user) the program computes star densities (i.e., nuclear interaction densities) as a function of location and particle type throughout the target. From these star densities, estimates of a number of quantities of radiobiological interest e.g., dose equivalent due to direct irradiation or due to exposure to remanent radioactivity are obtained.[4] At the user's option, the program also calculates momentum spectra of interacting particles, also as a function of location in the shield and of particle type.

A further option is the calculation of energy deposited by the cascade. The main assumptions underlying the mechanisms of energy deposition are described elsewhere.[4] This quantity is a useful measure of target heating and can be

related to light output of a plastic scintillator embedded in the target (ionization calorimeters). As presently written CASIM only yields the average energy deposited as a function of location. It should in principle be possible to study the fluctuations encountered in energy deposited per incident proton, albeit in less straightforward fashion than in conventional programs.

Alternative to the energy deposition option, the program calculates a relative error of the average (total) number of stars as a function of location. This is not a rigorous calculation and is only meant as a guide to the statistical validity of the answers. Further detail is given below (see V).

The accompanying version of CASIM is an attempt at a simple and broadly useful program, without deliberate biasing. As such it represents a conventional type calculation though more efficient.[1,2] A few geometries have been included which can be run without requiring any additional coding effort. Included also is an example on how the present version of CASIM may be modified to study a highly specific problem.

A note of caution in interpreting results: (1) Because of the way in which events are registered in the calculation there tends to be a high degree of correlation between neighboring volume bins. This may occasionally result in rather smooth results even when the calculation is patently statistically invalid. This is particularly true of neutrons which, regardless of their energy, create stars at large distances from their point of origin. (2) On the other hand, in common with all MC calcalations, there is a graininess when the limits of statistical accuracy are reached or exceeded. This is revealed more strongly by the charged particles where large fluctuations may be observed outside the core of the cascade. The user will have to bear this in mind when using the calculation directly as written. Far greater statistical accuracy can be achieved by special biasing at the expense of a lesser amount of general information.

Below, the alterations made to the Hagedorn-Ranft model are discussed. Following this is a brief chapter on the information to be supplied by a prospective user. The remainder is a description and listing of the program along with a glossary of the array and constant names. Finally, the specific program modification mentioned above is shown.

CASIM in written in the IBM 360, 370 Fortran IV language. Use of the H compiler with optimization OPT=2 results in considerable savings of computer time (~ 25%).

A listing of CASIM appears as a separate communication (FN-272. Supplement I) which includes also the subroutines which are changed to apply CASIM to a specific problem using special biasing (see XXVI).

I wish to thank M. Awschalom and P. Gollon for reading this manuscript and for discussion.

II. PARTICLE PRODUCTION MODEL

The particle production model employed in the present version of CASIM differs slightly from the one discussed previously.[4]

The particles considered in the calculation are protons (p), neutrons (n) and charged pions (π^+ and π^- are distinguished in the present version, although only the sums of the star densities due to π^+ and π^- are kept, this to reduce output volume). Neutral pions are not considered in propagating the cascade, but are very important in calculating energy deposition (see below). As stated above the particle production model used in CASIM is a set of inclusive distributions. The basis for these distributions is the Hagedorn-Ranft (HR) model.[5] A few additions and modifications are made:

(1) An extra set of distributions, representing relatively low energy nucleons knocked out of the nucleus, is added to the HR component. These distributions were taken from the parametrization by Ranft and Routti of Bertini's intranuclear cascade calculations.[6,7] To distinguish these particles from those derived from HR model distributions, they are henceforth called "slow" particles (vs "fast" particles, from HR).

(2) The HR model has so far been formulated only for pp collisions. Ranft has readjusted the free parameters to match 19.2 GeV/c p-nucleus data.[8,9] The following assumptions are made to represent the remaining processes.

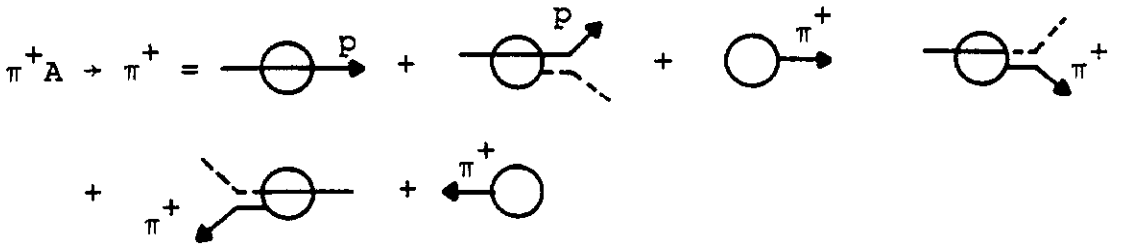
(a) Incident neutrons produce the same relative spectra of particles as do protons. Neutrons are produced with the same spectra as protons. The fractions of protons among all nucleons for different incident particles is assumed to be:

$$\begin{aligned} F(p) &= 0.5\ (1 + Z/A) \\ F(n) &= 0.5\ Z/A \\ F(\pi^{\pm}) &= Z/A. \end{aligned} \qquad \text{(II.1)}$$

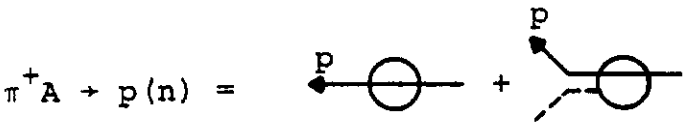
(b) The particle distributions for π induced reactions are derived by analogy with p induced distributions:

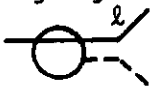
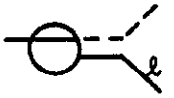
(i) The incident pion is replaced by a proton having the same center of mass (c.m.) momentum as the π in the πp frame.

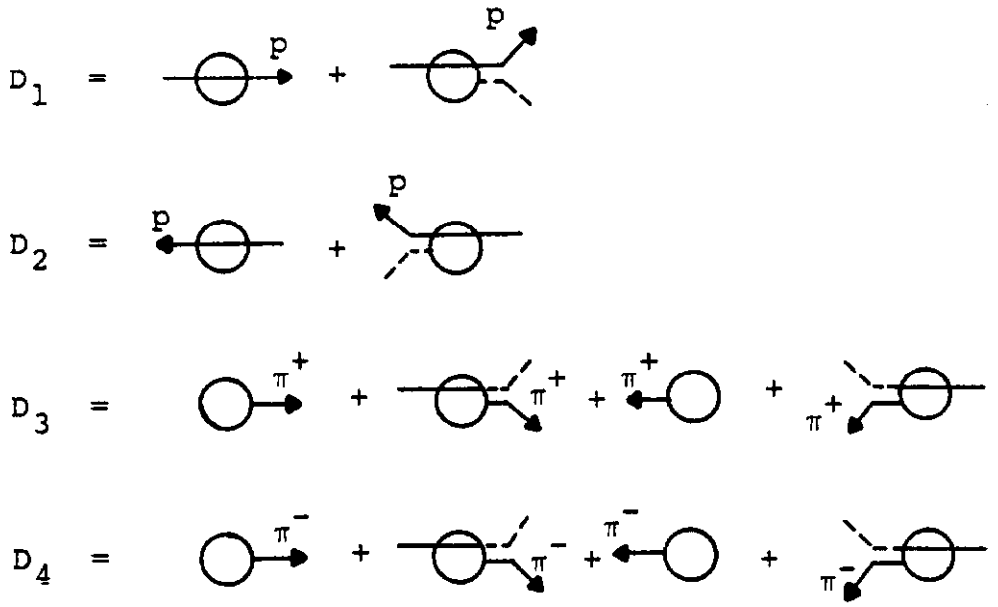
(ii) Only backward moving nucleons in the c.m. contribute to the outgoing nucleon spectra. Nucleons moving forward are assumed to be pions. Using Hagedorn's notation and for the incident pion moving left to right:



(II.2)



and similarly for π^-. Briefly,  represents an incident particle emerging from the collision, is a newly created particle,  and  are the decay products of low lying resonances (the label, ℓ, and the arrow indicate the particle for which the differential cross-section is calculated; the direction of the arrow indicates the direction of the particle in the c.m.). For more details, see Reference 5. For the reactions $\pi^+A \to \pi^-$ and $\pi^-A \to \pi^+$ the spectra of produced π's are taken to be those of the opposite charge produced in pA collisions. For the reactions $nA \to \pi^+$ and $nA \to \pi^-$ the spectra are assumed to be the same and equal to the average of $pA \to \pi^+$ and $pA \to \pi^-$. Note that the purpose of selecting fast particles during the MC calculation there are then four distributions from which to choose:



(II.3)

(c) The π° distributions, which are not explicitly given by HR are assumed to be the average of the non-leading π^+ and π^- (D_3 and D_4).

(d) The parameters for pion (and neutron) induced reactions are assumed to be the same as for proton induced reactions.

(e) The parameters of the distributions are those given in the "Atlas" of Grote, Hagedorn and Ranft.[5] For target nuclei other than those explicitly referred to in the "Atlas" (Be, Al, Cu, Pb) the parameters are obtained by interpolation.

(f) Each inelastic interaction creates a certain amount of nuclear excitation, E*, determined from a modified version of the formula used by Ranft,[10]

$$E^* = E' \qquad E_{inc} \geq 5 \text{ GeV}$$

$$E^* = 0.05 + \frac{(E_{inc}-0.05)}{4.95}(E'-0.05) \qquad 0.05 < E_{inc} < 5 \text{ GeV} \qquad \text{(II.4)}$$

$$E^* = E_{inc} \qquad E_{inc} \leq 0.05 \text{ GeV}$$

where E' = max (.005A, .05) and E_{inc} is the incident kinetic energy (for nucleons) or total energy (pions). In addition a binding energy correction of 8 MeV per (net) nucleon emitted is applied.

(g) The HR nucleon spectra are normalized to two outgoing nucleons (incident nucleons) or one outgoing nucleon (incident pions). If the energy balance does not permit this, the nucleon spectra are normalized so as to force energy conservation and π production is forbidden. If π production is kinematically forbidden the nucleon spectra are normalized to force energy conservation. A small adjustment is made to these assumptions in subroutine CALKI to simulate crudely the presence of kaons.

(h) The energy remaining after subtracting excitation and binding energy and energy carried off by nucleons (both HR and knock-out) from the incident energy is assumed to be carried off by pions. Their spectra are accordingly normalized.

The normalization factors are never far from unity, except at rather low energies ($\lesssim$ 3 GeV). A comparison of the HR model with a limited sample of data from p-nucleus experiments shows no gross discrepancies, at least insofar this would affect shielding calculations.[10] Most of CASIM is independent of the production model and changing to a different model should not cause great difficulties. Earlier, CASIM has been run with the Trilling formula[11,12] for hadron production.

III. USER SUPPLIED INFORMATION

The user is assumed to supply:

1. Some general information regarding the specific calculation submitted. This is read from two data cards (from MAIN):

NQINC ≡ type of incident particle (1=proton, 2=neutron, 3=π^+, 4=π^-).

PINC ≡ incident momentum in GeV/c.

PTR ≡ threshold momentum in GeV/c, below this momentum particles are no longer followed in the calculation.

BSIZ ≡ beam size. Beams are assumed to be square in cross section. BSIZ is the length of a side of the square in centimeter.

CLMU ≡ collision length multiplier. Should be equal to unity for most applications, but larger for problems of large dimensions. Caution: no convergence tests of CASIM have been run for large or small CLMU ($0.5 \lesssim$ CLMU $\lesssim 3$).

MAXST ≡ maximum number of particles (recording plus propagating) created in the calculation. This number is roughly proportional to the running time (minus the initialization).

A large run may have MAXST = 10^5 or even larger.

For debugging MAXST = 200 is a good choice.

ISCAT indicates whether to include elastic scattering (Coulomb and nuclear diffractive scattering); 0 = no, 1 = yes.

IEDEP indicates whether to calculate energy deposition

0 = no

1 = yes

2 = energy deposition only (this saves considerable running time).

IMOMS indicates whether to calculate momentum spectra

0 = no

1 = yes

Note that for IMOMS = 1, the user may wish to change all or part of the subroutines HPCAL and PSCAL (see below).

ISEED is any odd integer ($\leq$ 9 digits) for priming the random number generator.

WECO $\equiv$ lower bound on weight in the calculation, used for terminating both propagating and recording particles. A reasonable range is 10^{-9} - 10^{-5} depending upon the dimensions of the problem.

CASTEP $\equiv$ step length for determining distance to next collision (propagating particles) in centimeter. For most problems ~0.2 of the average collision length is a good choice.

RESTEP $\equiv$ step length used in recording the number of stars by a recording particle, in centimeter. This distance should generally be of the order of the volume bin dimensions.

For some problems it may optimize computer time to make CASTEP and RESTEP depend on the location (or material).

The above information is supplied on the first two data cards as follows:

NQINC, PINC, PTR, BSIZ, CLMU, MAXST, ISCAT
FORMAT (I10, 4F10.3, 2I10)
IEDEP, IMOMS, ISEED, WECO, CASTEP, RESTEP
FORMAT (3I10, E10.3, 2F10.3).

2. SUBROUTINE BINCAL and entries BINS, BINSTA, BINPT perform storing and output of information.

BINCAL initializes this program, BINS stores information during the MC run, BINSTA (if energy deposition is omitted) collects information during the run needed for a crude statistical analysis of the final (total) star densities and BINPT performs normalization, printing and plotting.

A default program is included in the accompanying version of CASIM. This program, in conjunction with the default program HITOR, treats one of three different geometries to be specified by the user (1) a solid cylinder (2) a target in a cylindrical cave and (3) a pipe with the beam interacting on the wall. In all three cases the output distinguishes protons, neutrons and pions. Case (3) may be run with an alternative option viz. a coarse azimuthal analysis of the total star density in the walls of the pipe. Note that the outer boundaries are always those of a cylinder. More detail on this version is given below. There are many problems which should require only relatively minor changes in this subroutine. Particularly the labor of recoding the entire section dealing with printing and plotting may be readily avoided. The sample geometries included can serve as illustrations.

The default version of BINCAL reads the following information from two data cards (which follows the data cards read from MAIN).

NGEOM ≡ type of geometry

1 = solid cylinder

2 = target in a cave

3 = pipe, particle type analysis

4 = pipe, coarse azimuthal angle analysis.

This is supplied on one data card:

NGEOM

FORMAT (I10).

Depending upon NGEOM the following information is read from the next card:

(a) NGEOM = 1.

ZLIM ≡ length of the cylinder (cm).

RLIM ≡ radius of the cylinder (cm).

ZIN ≡ distance from face of cylinder at which beam begins to interact (cm). ZIN $\neq$ 0 is useful to obtain some

information on albedo. A small positive ZIN (<< bin length) will protect against an incident particle being ruled outside the block.

Format:

ZLIM, RLIM, ZIN

FORMAT (3F103)

(b) NGEOM = 2.

ZLIM ≡ total length of cave (cm).

RLIM ≡ outer radius of cave (cm).

ZIN ≡ position of target face with respect to cave entrance (cm). ZIN > 0 is assumed.

TARL ≡ target length (cm).

TARR ≡ target radius (cm).

TUNL ≡ tunnel length, from entrance of cave (cm).

TUNR ≡ tunnel radius (cm).

Format:

ZLIM, RLIM, ZIN, TARL, TARR, TUNL, TUNR

FORMAT (7F10.3)

(c) NGEOM = 3 or 4.

ZLIM ≡ length of pipe (cm).

RLIM ≡ outer radius of pipe (cm).

ZIN ≡ distance from face of pipe where beam begins to interact (cm). The beam is assumed to be a point source at an infinitesimal distance inside the wall emitting beam particles travelling parallel to the pipe-axis.

TUNR ≡ inner radius of pipe (cm).

Format:

ZLIM, RLIM, ZIN, TUNR

FORMAT (4F10.3).

3. SUBROUTINE HITOR and entry HITORM supply respectively information regarding materials present in the calculation and their geometrical arrangement.

HITOR, called during initialization, assigns an index to each material present and provides (via text, data cards

or any other means) the following parameters (a) atomic number (ZT), (b) atomic weight (AT), (c) density (RHO) in g/cm^3, (d) critical energy (ECR) in MeV. The latter is used only when energy deposition is being studied.

In the default version of the program first the number of different materials is read.

Format:

NE

FORMAT (I10).

Next, ZT, AT, RHO and ECR are supplied on a separate data card for each element.

Format:

ZT(i), AT(i), RHO(i), ECR(i)

FORMAT (4F10.3).

Note that for ISCAT = 1 each such data card is followed by another data card specifying some parameters needed in evaluating elastic scattering.

Material index 0 is conventionally reserved for the vacuum and index 1 for the material struck by the beam.

NE = total number of different materials present

NOUT ≡ NE + 1 = index assigned to any location completely outside the geometric boundaries of the problem. The variables ZT, AT, RHO, ECR, NE, NOUT are stored in COMMON for use in other parts of the program. As presently written five different materials are allowed. This number can be increased by merely increasing the proper subscript in the arrays which depend upon the material.

HITORM relates the Cartesian coordinates X, Y, Z to material index. The sample geometries and the illustration of a dedicated program may serve as a guide.

4. SUBROUTINE SCAT and entry SCTR calculate multiple Coulomb scattering of charged particles and nuclear diffractive scattering. These effects are likely important only for problems of small dimensions. This option is activated by setting ISCAT = 1 in the first data card called from MAIN. No programming changes are needed but the user must supply

an extra data card for each element (which follows the data card read from HITOR in the default version). The following information is to be supplied:

RALG ≡ radiation length (cm).

RFM ≡ nuclear radius parameter (fermi).

SIGL ≡ nuclear elastic scattering cross section (barn).

The latter two may be taken e.g. from the paper of Belletini et al.[13]

Format:

RALG(i), RFM(i), SIGL(i)

FORMAT (3F10.3).

5. SUBROUTINE HPCAL and entries HPLOC and HPVOL are referenced only when momentum spectra of interacting particles are to be calculated (IMOMS = 1). HPCAL defines the volume bins in which the momenta of interacting particles are analyzed. Generally these should be much larger than the volumes used for calculating star densities to gain sufficient statistics in each momentum bin. HPLOC assigns the bin subscript given the Cartesian coordinates of the location and HPVOL supplies the bin volume during normalization.

   The default program divides the cylinder into 16 volume bins spanning four depth and four radial regions. More detail is given below.

6. SUBROUTINE PSCAL and entries PSCAM, PSPEK and PSPRT are also used only when calculating momentum spectra. PSCAL and PSCAM initialize the momentum-bin boundaries, PSPEK stores the information during the MC run and PSPRT normalizes and prints out the spectra. These need not be changed for different problems except that the dimensionality of the arrays PSPEC, ZPL, ZPH, RPL, RPH should conform to the dimensionality of the volumes defined in HPCAL. More detail is given below.

IV. MAIN PROGRAM

MAIN initializes the calculation and executes the propagation of the cascade. Star production by primary particles is also recorded from MAIN. (Star production by other than primary particles is recorded via the subroutine RECORD.)

Initialization starts with reading and printing the general parameters of the calculation including specification of the options: momentum spectra, star production, energy deposition and elastic scattering. Following this the quantities needed for the calculation are determined both directly in MAIN and by initialization of the various subroutines.

Many of the quantities needed in the calculation depend upon particle type. For some it is sufficient to distinguish between nucleons and pions, i.e., index NI = 1 (nucleons), 2 (pions). For others the distinction between protons and neutrons is kept i.e., NQF = 1 (p), 2 = (n), 3 = ($\pi^{\pm}$). If in addition π^+ and π^- must be distinguished, yet another index is called for i.e., NQI = 1 (p), 2 = (n), 3 = (π^+), 4 = (π^-).

Likewise, many of the quantities used depend upon the momentum of the particle. For most of these it has been found sufficient to store them in arrays in which the momentum varies between 0.1 GeV/c and 1000 GeV/c (the full momentum range of the calculation) logarithmically divided into 11 equal steps. For a specific momentum, the actual values of the quantities are determined by interpolation.

The quantities which depend on the material (atomic number and atomic weight) are initially stored in arrays for four "standard" materials (Be, Al, Cu, Pb). They are converted into equivalent arrays for the actual materials of the particular problem during initialization by means of interpolation.

During the Monte Carlo run a rough energy balance is kept by storing the energy lost into various "sinks" of the model:

(1) Losses by ionization (and bremssthralung and pair production) of charged hadrons.

(2) Energy carried off by particles below threshold momentum of the calculation, including both
   (a) particles emitted below threshold in nuclear collisions
   (b) charged particles which fall below threshold as they slow down.

(3) Energy carried off by hadrons escaping from the geometrical boundaries of the problem.

(4) Nuclear excitation energy.

(5) Energy carried off by neutral pions (which do not propagate the cascade).

(6) Energy of particles created below the weight cut-off of the calculation.

The sum of these sinks should in principle equal the total incident energy. In practice this holds true only within a few percent. The main reasons for this are (1) statistical uncertainties due to the finite duration of the calculation (2) errors in the normalization constants (introduced to enforce conservation of energy) which are obtained by numerical integration over the thermodynamical spectra (3) errors arising from interpolation. While it is clear that systematic errors are present the assumption is made that the deviation from unity is largely statistical. This is particularly so if a collision length multiplier either much larger or much smaller than unity is used. For this reason the fraction of the incident energy recovered is used to normalize the final results (in BINPT). This procedure is not recommended when a very high threshold is imposed on the calculation, unless steps are taken to evaluate the fractional energy of particles below threshold more accurately.

Before starting each cascade with a new primary particle a test is performed to see if NUMST exceeds MAXST. NUMST keeps count of the number of unweighted (propagating and recording)

particles generated and is a rough measure of elasped time, excluding initialization. Hence by specifying MAXST the user controls computer time spent. When NUMST exceeds MAXST, MAIN calls BINPT to perform normalization and printing out. Also at this point the program calls BINSTA every time a sub-run is completed i.e., when NUMST $\geq$ n· MAXST/10 (n = 1,2,...10) for the first time, for purposes of a crude statistical analysis (see V).

When a new propagating particle is generated (primary or other) the distance to the next collision, ℓ, is randomly determined in units of collision lengths, λ. For each step (of fixed length Δ) through which the particle is transported, ℓ is reduced by $(\Delta/\lambda_1 + \Delta/\lambda_2)/2$ where subscripts 1 and 2 denote the material at the beginning and at the end of the step (as determined by the subroutine HITORM). When ℓ becomes negative the particle is assumed to have undergone a collision during the previous step. To avoid systematic errors due to finite steplength, the first step, s, of the particle is chosen according to the probability distribution

$$P(s) = \exp(-s/c\lambda_1)/(c\lambda_1) \qquad 0 \leq s \leq \Delta, \tag{IV.1}$$

where λ_1 refers to the material where the particle is generated and c is the particular collision length multiplier used. Note that this may entail some error in a multiple media calculation. This will be small so long as $s \ll c\lambda$ for all media, or the collision lengths of the various media are not too disparate. Following each step HITORM is called to find the medium corresponding to the new coordinates. The following mutually exclusive possibilities occur:

(a) The particle is outside the problem boundaries. The difference in weighted energy of a charged particle at production and at escape is determined and stored in energy sink (1). The remainder is stored in (3). A new primary particle is generated.

(b) A charged particle falls below threshold momentum. The difference in energy at production and at threshold is added to (1). The energy at threshold is stored in (2). A new primary particle is generated.

(c) The particle underwent a collision. The energy difference of a charged particle at production and at collision is added to (1). The nuclear excitation energy of the collision is calculated and added to (4). The energy carried off by subthreshold particles and by neutral pions is obtained by interpolation from the arrays EBT and WIPO (calculated in CAMKI) and added to (2) and (5), respectively.

(d) None of the above. The particle is transported through the next step.

In the event (c) above, when the star density option is in effect, one or more recording particles are generated and the star densities along their trajectories are determined (in RECORD). However, if the particle is a primary particle, the star produced by this (propagating) particle is included in the star density at this point. A new propagating particle is then generated. If the weight of the particle is below the weight cut-off, the energy is added to sink (6) and a new primary particle is generated. Otherwise, the new particle now replaces the old particle. If the energy density option is in effect the subroutines EDP and (for non-leading pions) EMSHWR are called at this point. The weight of the pion in EMSHWR is adjusted for kaon production (see CALKI). The loop is completed and a new distance to collision is determined. An abbreviated flow chart of MAIN is shown in Fig. 1.

GLOSSARY OF ARRAY AND CONSTANT NAMES (MAIN)

I, J = auxiliary storage (a.s.).
V ≡ dummy argument in calling FUNCTION RAN.
X is used in interpolation.
W(i=1-4) ≡ mass of particle type i, in GeV (1=p, 2=n, 3=π^+, 4=π^-).
S = a.s.

AT(i=1-5) ≡ atomic weight of problem material with index i.
NA, NB = a.s.
ND ≡ distribution of outgoing particle. For ND $\leq$ 4 see (II.3), ND=5 denotes slow nucleon.
NE ≡ total number of different materials in problem.
NI ≡ type of incident particle (1 = nucleon, 2 = pion).
NM ≡ index of problem material.
XX is used in interpolation between tabular incident momenta.
XA, YA, ZA ≡ Cartesian coordinates of propagating particle (cm).
ZT(i=1-5) ≡ atomic number of problem material with index i.

ANG ≡ angle between incident and outgoing particle (radians).
BNG is ANG in double precision notation.
DCX, DCY, DCZ ≡ direction cosines of current propagating particle.
DEX, DEY, DEZ ≡ direction cosines of newly created propagating particle.
DPL ≡ logarithmic difference between tabular incident momenta
DUM ≡ a.s.
EBT(i=1-11, j=1-2, k=1-5) ≡ fraction of energy carried off by subthreshold particles in a collision with tabular incident momentum index i, of particle type j (1 = nucleon, 2 = pion) and nucleus of material with index k.
EFA, EFB, EST = a.s.
ETR(i=1-4) ≡ for nucleons, kinetic energy at threshold; for pions total energy at threshold (GeV); index i as in W.
JMM, JMX ≡ minimum and maximum index of incident momenta for which tables need to be prepared.
JMN ≡ a.s.

MUM ≡ a.s.

NPT is used in interpolation between tabular incident momenta.

NQF ≡ type of propagating particle (1=p, 2=n, 3=$\pi^{\pm}$).

NQI, NQO ≡ type of incident, outgoing propagating particle, see W.

NST, NUO = a.s.

PHI ≡ random azimuthal angle ($0 \leq \phi \leq 2\pi$), (radians).

PIN, QIN ≡ momentum of current propagating particle (GeV/c).

PLG ≡ logarithm of PIN.

PTN = a.s.

PTR ≡ threshold momentum of calculation (GeV/c).

RHO(i=1-5) ≡ density of problem material with index i (g/cm^3).

RIN ≡ range of current (charged) propagating particle (cm).

RTR(i=1-4, j=1-5) ≡ range at threshold of particle type i, see W in problem material with index j (cm).

TIM ≡ kinetic (nucleons) or total (pions) energy of propagating particle at point of collision or escape (GeV).

TIN ≡ kinetic (nucleons) or total (pions) energy of current propagating particle (GeV).

VPT is used in interpolation between tabular incident momenta.

XBT ≡ fraction of energy carried off by subthreshold particles upon collision of current propagating particle.

ZIN ≡ Z-coordinate of configuration at which beam begins to interact (cm).

ABSF(i=1-5, j=1-5, k=1-2) ≡ survival probability of recording particle during constant step (RESTEP) starting in material with index i and ending in j for particle type j, sec NI.

AMST(i=1-4) ≡ atomic weight of standard elements (1=Be, 2=Al, 3=Cu, 4=Pb).

AVMS ≡ average atomic weight of materials present in the calculation.

BSIZ ≡ beam size (cm). Beam is assumed to have a square cross section, BSIZ x BSIZ.

CAST(i=1-4, j=1-30) ≡ total number of cascade stars by type i (1=p, 2=n, 3=$\pi^{\pm}$, 4 = total) and generation number j.

CFCA(i=1-5, j=1-2) is used in determining the collision site within a step in material of index i, particle type j, see NI.

CLLL ≡ a.s.

CLMU ≡ collision length multiplier (used for neutrons and for charged particles with relatively high momentum).

COLT(i=1-5, j=1-2) ≡ collision length in problem material i, particle type j, see NI (cm).

DTOC ≡ distance to point of collision, in collision lengths.

ENCR(i=1-7) ≡ energy sink counters, index i (see text).

EXCT(i=1-5) ≡ average nuclear excitation energy resulting in the collision of a high energy particle ($p \geq 5$ GeV/c) with a nucleus of material type i (GeV).

MATE(i=1-5) ≡ atomic weight of problem material (index i) in fixed point notation.

MNEW, MOLD ≡ material index at beginning, end of step of propagating particle.

MZTE(i=1-5) ≡ atomic number of problem material (index i) in fixed point notation.

MFLG, NFLG indicate whether location of particle need be redetermined in storing event in momentum spectrum, star density counters (0 = yes, 1 = no).

NOUT ≡ index signifying location outside geometrical problem boundaries.

NPBI ≡ momentum bin of particle for momentum spectrum determination.

NPTR ≡ incident momentum bin corresponding to threshold momentum.

NREC ≡ number of recording particles selected by current propagating particle.

NUGN ≡ generation number of current propagating particle.

NUIN ≡ current number of primary particles.

PFCA(i=1-5, j=1-2), see CFCA.

PFRE(i=1-5, j=1-2) ≡ average number of stars produced within a recording step in material i, particle type j (see NI), per unit weight.

PINC ≡ momentum of primary particle (GeV/c).

POUS ≡ momentum of outgoing propagating particle in double precision notation (GeV/c).

POUT(i=1-2) ≡ momentum of outgoing particle (GeV/c). POUT (1) only is used for propagating particles.

PTRL ≡ log of threshold momentum, expressed in GeV/c.

RINC ≡ range of primary particle in material of index 1 (cm).

RLIM, see III.

SQMO ≡ weight factor incurred in selecting distribution of propagating particle.

TARL, TARR, see III.

TINC ≡ kinetic energy (for nucleons), total energy (for pions) of incident particle (GeV).

TUNL, TUNR, see III.

VFCA is used in determining location of collision site within propagating step.

VLMU ≡ collision length multiplier of current propagating particle.

WATE ≡ adjusted weight of pions representing π^0 production.

WECO ≡ weight cut-off of calculation.

WFAC ≡ weight factor incurred in selecting angle and momentum of propagating particle.

WGHT ≡ weight of current propagating particle.

WIPO(i=1-11, j=1-2, k=1-5) ≡ fraction of energy carried off by neutral pions in a collision with tabular incident momentum index i, of particle type j (1 = nucleon, 2 = pion) and nucleus of material with index k.

XIPO ≡ fraction of energy carried off by neutral pions in collision of current propagating particle.

ZLIM, see III.

COLST(i=1-4, j=1-2) ≡ collision length of particle type j (see NI) in g/cm^2 in standard material of index i as in AMST.

DPTRL is used in interpolation when incident momentum is in same bin as threshold momentum.

ESTAR ≡ nuclear excitation energy produced in collision of current propagating particle (GeV).

IEDEP indicates energy deposition option (0 = no, 1 = yes, 2 = only).

IMOMS indicates momentum spectrum option (0 = no, 1 = yes).

ISEED ≡ random number seed, any odd integer ($\leq$ 9 digits).

ISCAT indicates elastic scattering option (0 = no, 1 = yes).

KSTAT is used in determining when to call BINSTA for updating statistical analysis of all star density.

MATAV ≡ average atomic weight of materials present in the calculation (= AVMS, in fixed point notation).

MAXST ≡ maximum number of unweighted particles to be generated during calculation.

NFINC ≡ type of primary particle (1=p, 2=n, 3=$\pi^{\pm}$).

NGEOM ≡ index specifying type of geometry (see III).

NIINC ≡ type of primary particle (1 = nucleon, 2 = pion).

NQINC ≡ type of primary particle, see W.

NUMST ≡ current total number of unweighted particles generated.

PLMIN ≡ logarithm of lowest tabular value of incident momentum (0.1 GeV/c).

CASTEP ≡ step length used in determining distance to next collision of propagating particles (cm).

CBSTEP location of current propagating particle within CASTEP (cm).

NPBINC ≡ momentum bin of primary particle (see PSCAL).

RESTEP ≡ step length used in recording stars created by a recording particle (cm).

V. SUBROUTINE BINCAL (BINS, BINSTA, BINPT)

This program is normally supplied, at least in part, by the user. The version described here can be used as is only if it suits the geometry of the problem and will supply the information sought. Note that any changes in this subroutine may entail corresponding changes in the CALL statements from MAIN (for BINCAL, BINS, BINSTA and BINPT) from RECORD, EMSHWR and EDP (for BINS). For a different geometry as specified by HITOR changes may be needed in HPCAL and PSCAL.

The present version has built into it four different cases (see III). In BINCAL the type of geometry is specified and the dimensions describing it are read and printed. In BINS each event is analyzed by geometry (and particle type for cases 1-3) and stored. In BINSTA the necessary information for a crude statistical analysis of the all star densities is stored every time a sub-run (~ 1/10th of a run) is completed. BINPT performs normalization, printing and plotting.

For all cases the geometrical boundaries are those of a cylinder. The entire volume is divided into 50 x 50 array of adjoining volume bins delineated by 51 equally spaced depths and 51 equally spaced radii. For cases 1-3, stars due to protons, neutrons and charged pions are distinguished (π^+ and π^- are summed, solely to limit output volume). For case 1 the smallest radial region is further divided into five equi-spaced radii since the variation may be large there. For case 4 (beam striking the wall of a pipe) the distinction according to type of particle is dropped in favor of a crude azimuthal analysis of the total star density.

Note that when this general type of volume binning is used for a heterogeneous shield (such as cases 2-4) it is easier to interpret results when the material boundaries coincide with the volume bin boundaries. However, for case 2 (target in a cave) the present version yields the star density over the entire target (of arbitrary size) automatically.

When the energy deposition option is not in effect the program performs a crude statistical analysis on the total star density. For this purpose the entire MC run is divided into n sub-runs (n = 10 in the present version). As is well known the variance of the distribution of the results of the n sub-runs is estimated by

$$\sigma^2 = (\Sigma x_i^2 - n\langle x\rangle^2)/(n - 1), \qquad (V.1)$$

and the variance of the distribution of the mean

$$\sigma^2_{\langle x\rangle} = \sigma^2/n, \qquad (V.2)$$

where x_i is the star density at a particular location derived from the i^{th} sub-run and $\langle x\rangle$ is the star density at that location determined by the complete run. Printed out at the end is the relative error of $\langle x\rangle$, $r = \sigma_{\langle x\rangle}/\langle x\rangle$.

During the calculation the quantity Σy_i^2 is accumulated in BINSTA where y_i is the total number of stars at a given location during the i^{th} sub-run. The normalization is performed at the end in BINPT.

Note that the end of a sub-run is signalled by a test on the current number of unweighted stars rather than the current number of incident protons, since the eventual total number of primaries is unknown during the MC run. Hence, generally, $\langle x\rangle \neq \Sigma x_i/n$ although for a long run the difference should be small. Note also that $r \leq 1$, since the maximum of σ^2 will occur when $x_j \neq 0$ and all other $x_k = 0$ $(k \neq j)$, then $\Sigma x_i^2 = x_j^2 = n^2\langle x\rangle^2$ and substitution yields $r = 1$. This must be borne in mind when examining these relative errors.

BINPT is called from MAIN at the end of the MC run. Following normalization and, when applicable, calculation of the relative error BINPT prints:

(1) The (normalized) energy sink counters of the calculation (see IV).

(2) Number of stars per incident proton in the entire configuration for each type particle and summed over type, separately for propagating and recording particles.

(3) If the energy deposition option is in effect, the total energy deposited per incident proton in the entire configuration.

(4) Ditto as in (2), but in addition listed as a function of generation number.

(5) Star (energy) densities for the 50 x 50 (depth x radius) array. For cases 1-3 the star densities are printed separately according to type and summed over type. For case 4 the volume is further divided into three regions according to azimuthal angle but only the sum over type is kept.

(6) Relative error on total star density for the 50 x 50 array (in lieu of the energy density).

(7) Integrals of star (energy) density over radius and over depth for each type and summed over type (except in case 4, see above). Relative error on total star density is calculated in lieu of energy density.

(8) For case 1, star (energy) densities at small radius for a 50 x 5 (depth x radius) array.

BINPT plots for each type particle and for the sum over type the star density and/or the energy density:

(1) Integral over radius vs depth and integral over depth vs radius.

(2) The depth dependence at selected radii (10 radii, contained in two plots).

(3) The radial dependence at selected depths (15 radii contained in three plots).

(4) Contour plot of equal star density (summed over type only) and/or equal energy density.

BINPT also calls PSPRT which prints and plots the momentum spectra, if the option is in effect.

GLOSSARY OF ARRAY AND CONSTANT NAMES (BINCAL)

F(i=50, j=1-6), I, K ≡ auxiliary storage (a.s.).
R ≡ radial coordinate of event (cm).
W ≡ increment to be stored in counter.
X(i=1-51) ≡ a.s.

AR(i=1-5), IQ ≡ a.s.
IR ≡ radial index of volume bin. Also used as a.s. in BINPT.
IS ≡ a.s.
IZ ≡ axial index of volume bin. Also used as a.s. in BINPT.
JR ≡ radial index of volume bin (for small radii). Also used as a.s. in BINPT.
KR, LR ≡ a.s.
MQ ≡ azimuthal index of volume bin.
NG ≡ a.s.
NQ ≡ index of type of particle (1=p, 2=n, 3=$\pi^{\pm}$) or energy deposition (NQ=5).
XA, YA, ZA ≡ Cartesian coordinates of event (cm).

DSR(i=1-50, j=1-5) ≡ axial integral of star density for particle type j(1=p, 2=n, 3=$\pi^{\pm}$, 4=total, 5=relative error on 4, or energy density) and radial bin i [stars (or GeV)]/(cm)·(inc. p).
DSZ(i=1-50, j=1-5) ≡ radial integral of star density (see DSR).
IAR ≡ a.s.
IQH, IQL ≡ upper and lower limits of IQ for which to calculate or print arrays.
IQQ ≡ a.s.
RBL ≡ radial extent of volume bins (cm).
RCM(i=1-51) ≡ radial limits of volume bins of large array (cm).
RDM(i=1-6) ≡ radial limits of volume bins of small array (cm).
ZBL ≡ length of volume bins (cm).
ZCM(i=1-51) ≡ axial limits of volume bins (cm).

ZIN ≡ distance along Z-axis at which beam begins to interact (cm). The lower limit on Z of the configuration is taken as Z = 0 by convention.

CAST(i=1-4, j=1-30) ≡ number of propagating stars type i (1=p, 2=n, 3=$\pi^{\pm}$, 4=total) and generation number j.

DSCR(i=1-5, j=1-5) ≡ axial integral of star density of particle type j (1=p, 2=n, 3=$\pi^{\pm}$, 4=total, 5=energy density) and radial bin i (for small radii) in stars (or GeV)/(cm · inc.p).

DSTC(i=1-50, j=1-50, k=1-5) ≡ star density of particle type j (see DSCR), radial bin j and axial bin i (for small radii) in stars/(cm^3)(inc.p).

EDPT ≡ energy deposited in entire configuration (GeV).

ELVL ≡ volume of smallest volume bin of the large arrays (cm^3).

ENCR(i=1-7) ≡ energy sink counter for type i (see IV); i=7 is the sum of all energy sinks.

NFLG indicates whether to locate volume bin of event (=0) or to store in previously determined volume bin (=1).

NUGN ≡ generation number of recording particle.

REST(i=1-4, j=1-31) ≡ number of recording stars type i (see CAST) and generation number j.

RLIM ≡ radial limit of entire configuration (cm).

TARL, TARR ≡ (case 2 only) length, radius of target (cm).

TCST, TRST ≡ total number of propagating, recording stars in entire configuration.

TINC ≡ kinetic (nucleons), total (pions) energy of primary particle (GeV).

TUNL ≡ (case 2 only) length of tunnel (cm).

TUNR ≡ radius of tunnel (case 2-4), in cm.

ZLIM ≡ axial limit of entire configuration (cm).

DSTAR(i=1-50, j=1-50, k=1-5) ≡ star density of particle type k (see DSCR) in axial bin i and radial bin j in [stars(or GeV)]/(cm^3)·(inc.p).

IEDEP ≡ energy deposition option (0=no, 1=yes, 2=only).

IMOMS ≡ momentum spectrum option (0=no, 1=yes).

NGEOM ≡ geometry option (see III).

NUINC ≡ total number of primary particles incident on shield.

VNORM ≡ a.s.

WNORM(i=1-4) ≡ (case 4 only) adjusts for different volumes of azimuthal bins with identical radial index.

TARVOL ≡ (case 2 only) volume of target (cm^3).

VI. SUBROUTINE RECORD

RECORD traces the recording particle from its origin to removal by either (1) escape from the configuration (2) its weight becoming less than cut-off weight or (3) for charged particles, its momentum falling below threshold. Nuclear interactions are specifically excluded as a cause of removal of the recording particle. Instead this effect is included on the average i.e., for each step, ℓ, through which the recording particle is transported its weight is reduced by the factor $\exp(-\ell/\lambda)$, where λ is the collision length. Likewise the average number of stars (nuclear interactions) in the step is $[1-\exp(-\ell/\lambda)]$ per unit weight. The distribution of stars as a function of distance, x, within the step ℓ is assumed to be

$$P(x) = \lambda^{-1}\exp(-x/\lambda)/[1-\exp(-\ell/\lambda)] \qquad 0 \le x \le \ell \qquad \text{(VI.1)}$$

The following prescription is adopted: the distance x is determined from (VI.1) once per recording particle, at the start. The particle is first transported through a step of length x (at which point the range is updated) and then through a series of steps of length ℓ until removal. For charged particles, with a residual range R (i.e., range to removal) less than ℓ, at the beginning of a step, the star production per unit weight in the last step is taken to be $[1-\exp(-R/\lambda)]$.

The selection of particles and calculation of the weights are generally the most time consuming parts of CASIM. Therefore the computing efficiency is noticeably improved by tracking simultaneously those particles created by the same distribution D_k [where k=1-4 are the fast particles, see (III.3) and k=5 are the slow particles]. Such particles are assumed to differ only in an extra weight factor which is readily determined. However, when elastic scattering is to be computed (ISCAT=1), only a single particle is tracked. Also, since the selection probability of the

recording particle's momentum and angle is proportional to the differential yield and since for the fast particles symmetry in the c.m. is assumed, it follows that for any particle selected the particle with equal and opposite momentum will have the same weight. This is exploited by tracking both particles each with half their normal weight provided the second particle is above threshold (the originally selected particle is forced to be above threshold). If the second particle is below threshold only the original particle is tracked with full weight. The extra weight factor of 1/2 in the former case arises from the fact that selection function $S_j(p,\Omega)$ in (XXII.3) is then normalized to two instead of unity.

Likewise, since the particle's weight is independent of its azimuthal production angle, the efficiency is increased if the particle is tracked along different azimuthal angles, provided the polar angle of the particle is sufficiently large. Also for a particle produced by a primary particle close to the axis in a cylindrically symmetric geometry, there is little gain in splitting this particle over different azimuthal angles. The criterion used for determining the number of azimuthal angles is

$$n_\phi = \left[0.1w(\pi-\theta,\theta)_{min} + 1 \right], \qquad \text{(VI.2)}$$

where w is the weight of the recording particle and the heavy brackets denote truncation.

Finally, to improve statistics of the calculation, the cascade particles with large weight, w, are made to create more than one recording particle i.e.,:

$$n_R = \min \left(\left[w/30 + 1 \right], 3 \right), \qquad \text{(VI.3)}$$

where n_R is the number of recording particles. The above criteria have not been tested in detail for efficiency.

Note that in the present version π^+ and π^- are lumped together in the final output. This is sufficient for most applications and reduces output volume significantly. They are treated separately throughout the rest of the program, however, and to introduce the separation during tracking and recording requires relatively minor program changes.

GLOSSARY OF ARRAY AND CONSTANT NAMES (RECORD)

V ≡ auxiliary storage (a.s.).

KO ≡ index to denote removal of particle.
ND ≡ distribution of recording particle (ND=1-4 are fast particles, ND=5 are slow particles).
XA, YA, ZA ≡ Cartesian coordinates of propagating particle (cm).
XR, YR, ZR ≡ Cartesian coordinates of recording particle (cm).
XX is used in interpolation between tabular incident momenta.

ANG ≡ angle between incident (propagating) and outgoing (recording) particle (radians).
BNG is ANG in double precision notation.
DCX, DCY, DCZ ≡ direction cosines of propagating particle.
DEX, DEY, DEZ ≡ direction cosines of recording particle.
KAM(i=1-3) means (charged) recording particle of type i has been ranged out.
NPA(i=1-3), NPB ≡ current momentum bin index of recording particle type i.
NPT is used in interpolation between tabular incident momenta of the propagating particle.
NPX ≡ momentum index of recording particle at origin.
NQI ≡ type of propagating particle (1=p, 2=n, 3=π^+, 4=π^-).
NQL, NQH ≡ smallest and largest index of recording particles to be simultaneously tracked (1=p, 2=n, 3=$\pi^\pm$).
NQO ≡ type of recording particle selected (see NQI).
NQR ≡ type of the recording particle tracked, see NQL.
NUM ≡ indicates whether particle is the originally selected particle (=1), or the particle with opposite c.m. momentum (=2).
NUO ≡ indicates whether to track original particle only (=1) or in addition to track particle with opposite c.m. momentum (=2).
PGG ≡ a.s.

PHI ≡ azimuthal production angle of recording particle (radians).

PIN ≡ incident momentum of propagating particle (GeV/c).

REX ≡ absorption factor of recording particle over last step.

RGE(i=1-3) ≡ range of recording particle of type i (see NQL) at origin (cm).

RGF(i=1-3) ≡ residual range of recording particle of type i, see NQL (cm).

RGG ≡ a.s.

RTR(i=1-4, j=1-5) ≡ range at threshold momentum of particle type i (see NQI) in problem material j, in cm.

RWT(i=1-4, j=1-5, k=1-4, ℓ=1-5) ≡ weight of particle type k (see NQI), given selected distribution j (see ND) and incident particle type i (see NQI) for problem material ℓ.

TIN ≡ incident energy (kinetic-for nucleons, total-for pions) of propagating particle (GeV).

WRC(i=1-3) ≡ weight of recording particle type i (see NQL) at origin.

WRD(i=1-3) ≡ weight of recording particle type i (see NQL).

ABSF(i=1-5, j=1-5, k=1-2) ≡ survival probability of recording particle after a constant step (RESTEP) starting in material i and ending in j for particle type k (1 = n, 2 = π).

COLT(i=1-5) ≡ collision length in material i (cm).

IPHI ≡ a.s.

MATE(i=1-5) ≡ atomic number of problem material i, in fixed point notation.

MFLG, NFLG indicate whether location of particle need be redetermined in storing event in momentum spectrum, star density counters (0 = yes, 1 = no).

MNEW ≡ material index at origin of recording particle.

MREC ≡ a.s.

NNEW, NOLD ≡ material index at beginning, end of step of recording particle.

NOUT ≡ index signifying location outside geometrical problem boundaries.

NPHI ≡ number of azimuthal angles for which to track recording particle.

NREC ≡ number of recording particles to be selected.

NUGN ≡ generation number of propagating particle.

NUIN ≡ current number of primary particles.

PFRE(i=1-5, k=1, 2) ≡ average number of stars produced during a recording step in material i, for particle type j (see ABSF).

POUS ≡ momentum of particle at beginning of recording step (GeV/c), used only if elastic scattering is to be included.

POUT(i=1-2) ≡ momentum of originally selected recording particle (=1) and particle with opposite c.m. momentum (=2) (GeV/c).

SQMO ≡ weight factor incurred in selecting distribution of recording particle.

WECO ≡ weight cut-off of calculation.

WFAC ≡ weight factor incurred in selecting angle and momentum of recording particle.

WFCO ≡ weight cut-off adjusted for present material and number of recording particles to be tracked.

WGCO ≡ weight cut-off adjusted for material (at origin) and number of recording particles to be tracked.

WGHT ≡ weight of propagating particle.

WREC ≡ weight of selected recording particle at origin.

IMOMS indicates momentum spectrum option (0 = no, 1 = yes).

ISCAT indicates elastic scattering option (0 = no, 1 = yes).

MAXST ≡ maximum number of unweighted particles to be generated during calculation.

XSTEP, YSTEP, ZSTEP ≡ x, y, z components of step of recording particle (cm).

FTSTEP ≡ location of current particle within RESTEP (cm).

RESTEP ≡ step length used in recording stars by recording particle (cm).

VII. SUBROUTINE CARTID

CARTID plots the contours of equal star (energy) density corresponding to integral powers of ten. The aim is to provide a condensed picture of the results of the calculation. Considerable approximation is involved in the preparation of this graph and the printouts should be consulted for more accurate information.

To construct the contour labelled 10^n all quantities (e.g., star densities) $Q(i_z, i_r)$ for a given depth region i_z which satisfy

$$n-0.5 \leq \log_{10} Q(i_z, i_r) \leq n+0.5 \qquad (-19 \leq n \leq 10) \qquad \text{(VII.1)}$$

are given a weight

$$W(n,i_r,i_z) = (0.5 - |\log_{10} Q(i_z,i_r) - n|)\cdot(2i_r - 1) \qquad \text{(VII.2)}$$

where the first factor ensures that values of $Q(i_z, i_r)$ closer to 10^n are given more weight and the second factor is proportional to the volume of the bin. Those which do not satisy (VII.1) have $W(n, i_r, i_z) = 0$. The radius determined by

$$r(n,i_z) = \Sigma i_r \, W(n,i_r,i_z)/\Sigma W(n,i_r,i_z) \qquad \text{(VII.3)}$$

is then the point of the 10^n contour at depth i_z. Naturally, $r(n,i_z)$ must be rounded off for printer plotting. Note also that CARTID does not attempt to find the points belonging to the contour line closest to the radial boundary since this method would clearly underestimate these points.

It is obvious that CARTID is only useful for very simple geometries. For the ones included in CASIM a problem could arise for the case of a target in a cave since CARTID does not

recognize the three different media (target, vacuum and cave). This problem has been avoided by providing star density curves only in the cave. For more complex geometries no such simple expedient may be found, besides which this way of representing the results may not be very useful.

GLOSSARY OF ARRAY AND CONSTANT NAMES (CARTID)

F ≡ corresponds to array DSTAR in BINCAL (IQ identifies sub-array to be plotted.

I ≡ auxiliary storage (a.s.)

L(i=1-33) contains literal constants needed for labelling spaces and marking boundaries on the plot.

M ≡ a.s.

FL = $\log_{10} Q(i_z,i_r)$ + 20.5; the constant 20.5 insures FL to be positive.

IK, IL, IM ≡ a.s.

IQ (see F).

IR, IZ ≡ index of radial, longitudinal bins.

JR, JZ ≡ a.s.

LF ≡ index associated with the value of the contour line.

LM ≡ a.s.

CTR(i=1-30) ≡ $r(i,i_z)$, see (VII.3) but shifted by the constant 20.5 (see FL).

LVR(i=1-50) ≡ prints the label "RADIUS" of the vertical axis of the plot.

MAT(i=1-50, j=1-50) is the matrix containing the entries of the contour plot.

RCM(i=1-51), ZCM(j=1-51) ≡ radial, axial limits of bins (cm).

WEI = $0.5 - |\log_{10} Q - n|$, see (VII.3).

LFLO = lowest value of LF (at a given depth). The point of this contour line is not calculated.

WGTH = $\Sigma W(n,i_r,i_z)$, see (VII.3).

VIII. SUBROUTINE COFMT

This subroutine which is called from FPCAL provides some parameters of a pp collision in the c.m. given the identity and the momentum of the incident particle in the lab and the identity of the outgoing particle. For completeness the kinematical derivations are reproduced below.

For the purposes of evaluating the yields of the various processes an incident pion is replaced by an incident proton having the same c.m. momentum. The variable transferred to SPULAL is the total c.m. energy. For a πp collision, s, the square of this quantity is

$$s = m_p^2 + m_\pi^2 + 2m_p \, \varepsilon_\pi^{lab}, \tag{VIII.1}$$

where ε_π^{lab} is the total energy of the π in the lab and m_p, m_π are the proton and pion rest masses. Alternatively, expressed in terms of c.m. quantities

$$s = (\varepsilon_p^{cm} + \varepsilon_\pi^{cm})^2 = \varepsilon_p^{cm^2} + \varepsilon_\pi^{cm^2} + 2\varepsilon_\pi^{cm} \, \varepsilon_p^{cm}, \tag{VIII.2}$$

or

$$s = 2\varepsilon_p^{cm^2} + m_\pi^2 - m_p^2 + 2\varepsilon_\pi^{cm} \, \varepsilon_p^{cm}. \tag{VIII.3}$$

If the pion is replaced by a proton with momentum p^{cm} the total c.m. energy denoted by $\sqrt{s}'$ is equal to $2\varepsilon_p^{cm}$.
From (VIII.2)

$$\sqrt{s} \, \sqrt{s}' = 2\varepsilon_p^{cm^2} + 2\varepsilon_p^{cm} \, \varepsilon_\pi^{cm} = s - m_\pi^2 + m_p^2, \tag{VIII.4}$$

and

$$\sqrt{s}' = (s - m_\pi^2 + m_p^2)/\sqrt{s}. \tag{VIII.5}$$

The lab energy of a proton, ϵ_p^{lab}, with c.m. momentum p^{cm} is obtained from (VIII.1). The Lorentz factors γ and $\eta = \beta\gamma$, where β and γ are the velocity (in units of c) and the total energy (in units of the restmass), are

$$\gamma = (\epsilon_p^{lab} + m_p)/\sqrt{s}' \qquad \text{(VIII.6)}$$

$$\eta = p_p^{lab}/\sqrt{s}' \ . \qquad \text{(VIII.7)}$$

The c.m. momentum is obtained by transforming the target proton ($p^{lab} = 0$) to the c.m.

$$p^{cm} = \eta m_p \ . \qquad \text{(VIII.8)}$$

Finally, the maximum c.m. energy and momentum of pions produced in pp collisions is calculated. This occurs when a single pion is produced and the two protons have equal momentum and travel parallel to each other:

$$s' = 4\epsilon_{0,p}^{cm^2} + \epsilon_{0,\pi}^{cm^2} + 4\epsilon_{0,p}^{cm}\ \epsilon_{0,\pi}^{cm} \qquad \text{(VIII.9)}$$

or

$$s' - 4\epsilon_{0,p}^{cm^2} + \epsilon_{0,\pi}^{cm^2} = 2\epsilon_{0,\pi}^{cm}\ (\epsilon_{0,\pi}^{cm} + 2\epsilon_{0,p}^{cm}), \qquad \text{(VIII.10)}$$

and (VIII.10) can be re-written as

$$s' - 4m_p^2 + m_\pi^2 = 2\epsilon_\pi^{cm}\ \sqrt{s}' \ . \qquad \text{(VIII.11)}$$

GLOSSARY OF ARRAY AND CONSTANT NAMES (COFMT)

W(i=1-4), WS(i=1-4) ≡ mass, mass squared (GeV, GeV^2) of particle type i (1=p, 2=n, 3=π^+, 4=π^-).

CME ≡ total energy in c.m. of pp collision (GeV).
GCM ≡ Lorentz factor γ of c.m. in lab.
NQI, NQO ≡ index of incident and outgoing particle (see W).
PIN ≡ incident momentum in lab (GeV/c).

CMEP ≡ square of total c.m. energy in collision of incident particle with a proton (GeV^2).
CMES = square of CME (GeV^2).
ECMX, PCMX ≡ max. energy, momentum of outgoing particle in c.m. (GeV, GeV/c).
EEQL, PEQL ≡ energy, momentum of a proton in lab with c.m. momentum equal to that of incident particle (GeV, GeV/c).
ETCM ≡ Lorentz factor $\eta(=\beta\gamma)$ of c.m. in lab.
PICM ≡ momentum of colliding particles in c.m. (GeV/c).

IX. SUBROUTINE SCAT (SCTR)

SCAT initializes and SCTR calculates the change in direction cosines due to (1) multiple Coulomb scattering of charged particles (2) nuclear diffractive scattering, in each step. It is assumed that the steps taken are sufficiently small that (1) multiple nuclear diffractive scattering can be neglected and (2) the displacement suffered by a particle within a step may be ignored. This is consistent with the assumption that these effects are only important for problems with small dimensions.

In SCAT the radiation length (in cm), the nuclear radius (in fermi) and the total elastic scattering cross section (in barn) are read in from data cards.

In SCTR the direction cosines at the end of the step are calculated from the direction cosines at the start of the step given the momentum, range (for charged particles) and the step length.

The prescription used to calculate the deflection for multiple Coulomb scattering is based on the treatment of Fermi outlined by Rossi.[14] It can readily be shown that if the distribution of projected angles is Gaussian, the corresponding distribution of θ_c^2, the square of the polar angle after traversal of a thickness Z of the material is

$$P_c\ (Z,\theta_c^2) = (1/\theta_s^2 Z)\ \exp\ (-\theta_c^2/\theta_s^2 Z), \qquad \text{(IX.1)}$$

where $\theta_s = (E_s/\beta p\sqrt{X_0})$. E_s is a constant ($\simeq$21 MeV), β and p are the velocity and momentum of the particle and X_0 is the radiation length.

Likewise, for nuclear diffractive scattering the optical model predicts a similar distribution

$$P_N\ (Z,\theta_N^2) = (Z/\lambda_{el})(4/R^2p^2)\ \exp\ (-R^2p^2\theta_N^2/4), \qquad \text{(IX.2)}$$

where λ_{el} is the mean free path for elastic scattering and R is the nuclear radius.

If the small angle approximation, already used in deriving equations (IX.1 and 2), is again invoked, then

$$\theta^2 = \theta_C^2 + \theta_N^2 \qquad \text{(IX.3)}$$

is the total scattering angle.

GLOSSARY OF ARRAY AND CONSTANT NAMES (SCAT)

I ≡ auxiliary storage (a.s.).
P ≡ momentum of particle (GeV/c).
R ≡ range of (charged) particle in material NM (cm).
V ≡ a.s.
W(i=1-4) ≡ mass of particle type i (1=p, 2=n, 3=π^+, 4=π^-) in GeV.

AT ≡ atomic weight of material(s) during initialization.
DX,DY,DZ ≡ direction cosines of particle.
EX, EY, EZ ≡ a.s.
NM ≡ material index.
NQ ≡ type of particle (see W).
PP ≡ a.s.

BNG ≡ polar scattering angle incurred during step (radians).
PHI ≡ random azimuthal angle (radians).
RFM(i=1-5) ≡ nuclear radius of material i (fermi).
RHO ≡ density of material(s) during initialization (g/cm^3).
THS ≡ square of BNG (radians^2).

BETS ≡ geometric average of the square of the velocity of the particle.
RALG(i=1-5) ≡ radiation length of material i (cm).
SIGL(i=1-5) ≡ elastic scattering cross section of material i (barns).
STEP ≡ propagating - or recording step for which scattering angle is to be evaluated (cm).

ECOEF(i=1-5), ECOLI(i=1-5) ≡ a.s.

X. FUNCTION RAN

Yields a random number between 0 and 1. This is the same as subroutine RANDU from the IBM Scientific Subroutine Library. It depends on the computer being used.

XI. SUBROUTINE PSCAL (PSCAM, PSPEK, PSPRT)

PSCAL and PSCAM perform initialization, PSPEK bins and stores interacting (recording) particles according to momentum and PSPRT performs normalization, printing and plotting.

PSCAL defines the limits of the bins as follows:

Bin Number	Momentum Range (GeV/c)
1	$10^{-1.05} - 10^{-0.95}$
2	$10^{-0.95} - 10^{-0.85}$
...	...
41	$10^{2.95} - 10^{3.05}$

Generally, the incident momentum and threshold momentum do not coincide with the bin limits. The correction, to be applied in the normalization, is calculated in PSCAL.

PSCAM translates the momentum bin limits into residual ranges (range to where a charged particle reaches threshold momentum). This facilitates the momentum binning while tracking a charged recording particle.

In PSPEK the volume bin of the location is found by calling HPLOC. In this version of the program, locations inside the boundaries of the problem but where no momentum analysis is to be performed are assigned the index NZ = 5 ($1 \leq NZ \leq 4$, otherwise). In RECORD the index NPB, the momentum bin of the particle at birth is calculated. For neutrons this index remains unchanged. For charged particles NPB varies as the particle loses momentum. At each recording step the program searches for the momentum bin starting from NPB downward. The new momentum bin index then gets stored in NPB.

The quantity MFLG indicates whether the volume bin of the collision needs to be determined (MFLG = 1) or the event can be stored in the previously determined volume bin (MFLG = 0).

Note that in the present version for NGEOM = 4 the momentum analysis is still performed by particle type and therefore will yield an average over azimuthal angle.

PSPRT performs the normalization and output. Some variables referenced in PSPRT are stored in COMMON/HPS/which provides a link with HPCAL where the volume bins are defined. The user wishing to redefine these arrays (as to number or number of subscripts) must make sure this is performed consistently in both programs.

GLOSSARY OF ARRAY AND CONSTANT NAMES (PSCAL)

F(i=1-50, j=1-5), I, J, N, X(i=1-50) ≡ auxiliary storage (a.s.).
R ≡ range of the recording particle (cm).

AR(i=1-5), IQ ≡ a.s.
NM ≡ material index.
NP ≡ momentum bin index of particle.
NZ, NR ≡ indices defining volume bin (large bins used for the momentum spectra).
WT ≡ weight of recording particle.
XA, YA, ZA ≡ Cartesian coordinates of recording particle (cm).

NPB ≡ momentum index of the previous recording (or birth); for neutrons this is constant throughout tracking.
NQI ≡ index of type of particle (1=p, 2=n, 3=$\pi^{\pm}$). For NGEOM = 4, case of a pipe with azimuthal angular analysis this index signifies the azimuthal angular quadrant (1 = struck by beam, 2 = at right angles, 3 = opposite).
PIN ≡ momentum of primary particle (GeV/c).
PTR ≡ threshold momentum of calculation (GeV/c).
RPL(i=1-4, j=1-4), RPH(i=1-4, j=1-4), ZPL(i=1-4, j=1-4), ZPH(i=1-4, j=1-4) ≡ lower, upper limits of radii, depths of volume bin i, j (cm).
RTR(i=1-4, j=1-5) ≡ range at threshold of particle type i (1=p, 2=n, 3=π^{+}, 4=π^{-}) in material with index j (cm).
VOL ≡ volume of bin (cm^3) denoted by NZ, NR.
ZIN ≡ distance at which beam begins to interact (cm). The lower limit on Z of the configuration is taken as Z = 0 by convention.

MFLG indicates whether to redetermine volume bin or store event in previously determined volume bin.
NINC ≡ total number of incident primaries.

PBIN(i=1-42) ≡ limits of momentum bins (GeV/c); PBIN (i) is the lower limit of bin i and upper limit of bin (i + 1).

PWID(i=1-41) ≡ width of the i^{th} momentum bin adjusted for incident-and threshold momentum (GeV/c).

RBIN(i=1-42, j=1-3, k=1-5) ≡ residual range of (charged) particle type j (see NQI), with momentum equal to PBIN (i) in material with index k (cm).

PSPEC(i=1-41, j=1-4, k=1-4, ℓ=1-4) ≡ momentum spectrum i.e., particles/$(cm^3)\cdot$(GeV/c)$\cdot$(inc. particle) in momentum bin i of particle type ℓ (see MQI, including comment about NGEOM = 4) in volume bin denoted by depth and radial indices j, k.

XII. SUBROUTINE PLOT

This subroutine plots a semi logarithmic graph of

(1) Radially integrated star (energy) density vs depth

(2) Star (energy) density vs depth at fixed radius

(3) Auxiliary integrated star (energy) density vs radius

(4) Star (energy) density vs radius at fixed depth.

If IMOMS = 1, this subroutine further provides:

(5) Momentum spectra at various locations (on a full logarithmic plot). The abcissa for cases (1-4) spans the full 50 bins (depth or radius) of the star density arrays. For case 5 there are 41 locations (10^{-1} to 10^{3} GeV/c, inclusive). The ordinate in all cases spans 12 decades which are chosen according to the highest value to be plotted.

For cases (2) and (4) the program plots the result for five different depths and radii, respectively, on the same graph, using different symbols (*, 2, 3, 4, 5). These symbols are given locations on the graph in the order they appear above. Therefore where two or more symbols should appear simultaneously only the last one will be printed.

For case (5) the program plots the momentum spectra of protons (*), neutrons (2), charged pions (3) and all stars (4) for a given volume bin on the same graph. The above comment about multiple symbols at the same location applies here also.

GLOSSARY OF ARRAY AND CONSTANT NAMES

F(i=1-50, j=1-5) ≡ array to be plotted.
I, M, N ≡ auxiliary storage.
L(i=1-13) contains literal constants needed for labelling spaces and marking boundaries on the plot.
X(i=1-51) labels the abcissa.
Y(i=1-13) labels the ordinate.

AR(i=1-5) labels the caption for multiple plots.
II, IM, IN, IP ≡ a.s.
NQ ≡ type of particle (1=p, 2=n, 3=$\pi^{\pm}$, 4=all stars), used to label heading.

IDX ≡ type of plot (see text).
IMX ≡ number of spaces for abcissa.
RHI, RLO, ZHI, ZLO ≡ upper, lower boundaries (radii, depths) of volume bin for momentum spectra (cm).
YMX ≡ highest (power of 10) label of ordinate.

LINE(i=1-121) contains the information of a line of the plot (corresponding to one space on the abcissa).

NPLOT ≡ number of different plots appearing on same graph.

XIII. SUBROUTINE RMCAL (ENTRY RGIP, PGIR)

This subroutine relates range in the problem materials to momentum. RMCAL initializes, RGIP calculates the range (cm), given the momentum (GeV/c) and PGIR calculates the momentum given the range (plus material index and type of particle).

RMCAL contains a table of proton ranges (expressed in g cm^{-2}) for each of 51 momenta logarithmically spaced between 0.1 and 1000 GeV/c and for each of the four standard elements (Be, Al, Cu, Pb). In RMCAL corresponding tables are prepared for the problem materials by linear interpolation over the atomic weight.

In RGIP and PGIR, the ranges or momenta are then obtained by linear interpolation over the momentum or range, respectively. For particles of unit charge other than protons, the ranges and momenta are obtained from the proton ranges using the familiar scaling relation

$$R_i(p_i) = (m_i/m_p)R_p(m_p p_i/m_i), \qquad \text{(XIII.1)}$$

where R_i, p_i and m_i are the range, momentum and mass of particle type i.

The range-momentum relations for the standard elements are based on compilations of Barkas and Berger[15] and Theriot.[16]

GLOSSARY OF ARRAY AND CONSTANT NAMES (RMCAL)

I, J ≡ auxiliary storage (a.s.).
P ≡ momentum of particle (GeV/c).
Q ≡ a.s.
R ≡ range of particle (cm).

FQ(i=1-4) ≡ scaling factor (see XIII.1) for particle type i
(1=p, 2=n, 3=π^+, 4=π^-).

IM ≡ a.s.
NM ≡ index of problem material.
NQ ≡ index of particle type (see FQ).
PL(i=1-51) ≡ table of logarithmically spaced momenta between 0.1 GeV/c and 1000 GeV/c.
XX is used for interpolation.

DPL ≡ logarithmic spacing between tabular momenta PL.
MZT ≡ atomic number of problem material.
RLM(i=1-51, j=1-5) ≡ range (cm) of proton in problem material j with momentum PL(i).

DENS ≡ density of problem material (g/cm^3).
NZST(i=1-4) ≡ atomic number of standard material with index i.
RLST(i=1-51, j=1-4) ≡ range (g/cm^2) of proton in standard material j with momentum PL(i).

PLMIN ≡ log (0.1), which is the lowest momentum in table PL(i).

XIV. SUBROUTINE CALKI (CAMKI, SELKI)

CALKI and CAMKI initialize the general and material dependent parts of the program. SELKI performs the selection of particle type and particle distributions (for fast particles) and calculation of the weight factor.

In CALKI two large arrays are stored. One array contains the normalization factors which multiply the production yields of the HR model (see II). These factors depend upon incident momentum (represented by 11 different momenta, logarithmically spaced between 0.1 and 1000 GeV/c, as elsewhere in the program), upon the distribution of outgoing particle (but they are identical for the distributions D_1 and D_2 as well as for D_3 and D_4), upon the type of incident particle (but only whether a nucleon or pion) and upon the target nucleus (represented by the four standard materials Be,Al,Cu,Pb). The other array contains the multiplicities and inelasticities of the fast particles, and depends upon incident momentum, distribution of outgoing particles, type of incident particle (nucleon or pion) and target nucleus. A small adjustment to these quantities is made in CAMKI. This is to include in first approximation the presence of kaons.

In CAMKI the normalization factors, multiplicities and inelasticities are calculated specifically for the material(s) present in the program. The multiplicities and inelasticities are then adjusted for calculational threshold. This is done by subtracting the subthreshold multiplicities and inelasticities from the listed ones. To save time the integration to calculate these subthreshold quantities is performed rather crudely. As such it is quite satisfactory for low thresholds. Care must be taken when the threshold is rather large to ensure that the corrected multiplicities and inelasticies are reasonably well calculated. Note however that this only affects the selection functions and hence should only influence the convergence properties of the calculation. When the momentum of the colliding particles is such that nucleons emitted backward in the lab are

always below threshold the multiplicities and inelasticities of the distribution D_2 are set identically to zero. At the same time, the fraction of the energy carried off by subthreshold particles in a collision is calculated and stored in tables for use in MAIN.

The arrays VNORM and VMI are calculated using the assumptions listed in III. To simulate crudely the presence of kaons, the arrays VMI, for the distributions of non-leading pions (ND=3,4) are multiplied by a factor of 1.059. This factor arises in the following way: let $A(\pi^+)$ and $A(\pi^-)$ be the multiplicities and inelasticities represented by VMI corresponding to ND=3,4 and for incident momenta above 3 GeV/c. Call B(i) the actual multiplicity or inelasticity of particle type i. Then assuming

$$\begin{aligned} B(\pi^\circ) &= 1/2\ B(\pi^+) + 1/2\ B(\pi^-) \\ B(K^\pm) &= 0.1\ B(\pi^\pm) \\ B(K^\circ) &= B(\bar{K}^\circ) = 0.1\ B(\pi^+) + 0.1\ B(\pi^-), \end{aligned} \tag{XIV.1}$$

and $\Sigma B(i) = 3/2\ [A(\pi^+) + A(\pi^-)]$ leads to

$$1.7\ [B(\pi^+) + B(\pi^-)] = 1.5\ [A(\pi^+) + A(\pi^-)], \tag{XIV.2}$$

or

$$B(\pi^\pm) = 0.882\ A(\pi^\pm). \tag{XIV.3}$$

If it is assumed that all kaons interact rather than decay, and if $C(\pi^\pm)$ represent the multiplicity or inelasticity of a combination of particles, viz.,

$$C(\pi^\pm) = B(\pi^\pm) + B(K^\pm) + 1/2\ B(K^\circ) + 1/2\ B(\bar{K}^\circ) \tag{XIV.4}$$

then, combining (XIV.3 and 4) yields

$$C(\pi^\pm) = 1.2\ B(\pi^\pm) = 1.059\ A(\pi^\pm). \tag{XIV.}$$

The justification for this procedure lies in the assumption

that the effects of the kaons (e.g. in dosimetry) are similar to those of pions.

Also in CAMKI, the inelasticity of π° are calculated for use in MAIN. These are likewise adjusted using the above assumptions

$$B(\pi^\circ) = 0.882\ [A(\pi^+) + A(\pi^-)]/2, \qquad \text{(XIV.6)}$$

or

$$B(\pi^\circ) = 0.833\ [C(\pi^+) + C(\pi^-)]/2. \qquad \text{(XIV.7)}$$

This last factor is also used in MAIN to calculate the weight of π° for use in EMSHWR.

The fractional multiplicity and inelasticity of slow particles (above threshold) are then calculated in CAMKI. Finally, the probabilities for selecting outgoing particle type, given incident particle type, distribution of outgoing particle and material are computed.

In SELKI the distribution of the particle is decided using a probability cut-off of 0.005 i.e. if for a particular distribution the fractional distribution is less than 0.005 the particles belonging to that distribution are neglected. This probability cut-off is introduced to avoid undue fluctuations, particularly in relatively short MC runs. As mentioned above the selection probabilities are not calculated with great accuracy, particularly when they become small. This could result in significant undersampling and hence in fluctuations. (Alternatively, one could resort to deliberate oversampling. However, it appears that most particles which are neglected by the probability cut-off can arise in many different ways in the calculation. Therefore neglecting a relatively rare mode of production should not have a large effect). The user is advised that the actual numerical value for the cut-off is rather arbitrary and its effect on the calculation has not been studied. Similar problems arise in the momentum and angle selection of fast particles, see (XXIV).

SELKI is called from MAIN every time a propagating particle undergoes an inelastic nuclear collision, except when the incident

particle is a nucleon below 50 MeV kinetic energy. It is called once to generate a propagating particle and (if applicable) once for each recording particle.

First it is decided whether the particle is fast or slow. For a fast particle the distribution is first decided, then the type. The weight factor is calculated in stages

$$W = N_{HR} \, [S \text{ (fast)} \cdot S \text{ (distribution)}]^{-1}, \qquad \text{(XIV.8)}$$

where N_{HR} is the normalization factor introduced in the HR model (see above), S (fast) is the selection probability for a fast particle and S (distribution) is the selection probability of the particular distribution chosen. No weight factor for type is explicitly included because (a) the HR yield as calculated in SUBROUTINE SPULAL (SPULAM, SPUL) is the sum over types of each distribution, (b) the selection of type is completely unbiased. Stated differently, the factors multiplying the HR yield to take into account the type always cancels the extra weight factor incurred in the selection of type. For a slow particle the weight factor is simply

$$W = S^{-1} \text{ (slow)}, \qquad \text{(XIV.9)}$$

since there is only a single distribution and no extra normalization is needed. Also in this case no weight factor for type need be introduced.

GLOSSARY OF ARRAY AND CONSTANT NAMES (CALKI)

A ≡ atomic weight of problem material.

I, J, K, V ≡ auxiliary storage (a.s.).

W(i=1-4) ≡ mass of particle type i (1=p, 2=n, 3=π^+, 4=π^-) in GeV.

X, Y ≡ integration variables x=exp ($-kp_T^2$) where k is the variable SLO and y=$p_Z^{cm}/p_{Z,max}^{cm}$.

Z ≡ atomic number of problem material.

DC ≡ dN/dxdy (normalized).

DX, DY ≡ integration intervals of x,y.

EW ≡ maximum energy in the lab of nucleon moving backward in c.m. (GeV).

JP, JX, JY ≡ a.s.

KD ≡ index of outgoing particles (see W).

NA, NB, NC ≡ a.s.

ND ≡ index of distribution (1-4 see II, 5=slow particle).

NF ≡ index of incident particle (2=nucleon, 4=pion).

NI ≡ index of incident particle (1=nucleon, 2=pion).

NK, NT ≡ a.s.

NM ≡ index of problem material.

PT, PZ ≡ p_T and p_Z^{cm} (GeV/c).

SL, SN ≡ a.s.

XX ≡ used in interpolation.

AST(i=1-4) ≡ atomic weight of standard materials i (1=Be, 2=Al, 3=Cu, 4=Pb).

BTI ≡ energy carried off by particles (p, n, $\pi^\pm$) below threshold (GeV).

BTM ≡ multiplicity of particles (p, n, $\pi^\pm$) below threshold.

CME ≡ total energy in c.m. of pp collision (GeV).

DPL ≡ logarithmic spacing between tabular momenta (0.1 - 1000 GeV/c).

EBT(i=1-11, j=1-2, k=1-5) inelasticity of particles below threshold in the collision of particle with momentum index i (see DPL), type j (see NI) in problem material k.

ETR(i=1-4) ≡ total energy of particle type i (see W) at momentum threshold (GeV).

GCM ≡ Lorentz factor γ of c.m. in lab.

JMM, JMX ≡ minimum and maximum tabular momentum index for which to evaluate arrays.

MAT ≡ a.s.

NMI ≡ index referring to multiplicity (=1) or inelasticity (=2).

NQI, NQO ≡ type of incident, outgoing particle (see W).

PIM ≡ tabular momentum (see DPL), in GeV.

PIN ≡ momentum of incident particle (GeV/c).

PTR ≡ threshold momentum (GeV/c).

PTS ≡ square of PT $(GeV/c)^2$.

RWT(i=1-4, j=1-5, k=1-4, ℓ=1-5) ≡ weight of particle type k (see W) given selected distribution j (see ND) and incident particle type i (see W) for problem material ℓ.

SLO ≡ parameter k defining the transverse variable x $(GeV/c)^{-2}$.

TIN ≡ kinetic (nucleons) or total (pions) energy corresponding to PIM (GeV).

TTR ≡ kinetic energy of nucleons at threshold momentum (GeV).

VMI(i=1-11, j=1-4, k=1-2, ℓ=1-4, m=1-2) ≡ multiplicity (m=1) or inelasticity (m=2) of distribution j (see W) for incident particle of momentum index i (see DPL) type k (see NI) in standard material ℓ (see AST).

VSF(i=1-11, j=1-2, k=1-5, ℓ=1-2) ≡ fractional multiplicity (ℓ=2) of slow nucleons for incident particle type j (see NI) of momentum index i (see DPL) in problem material k.

WMI(i=1-11, j=1-4, k=1-2, ℓ=1-5, m=1-2) ≡ multiplicity or inelasticity in problem material ℓ (other indices as in VMI above).

WNR ≡ normalization factor of particle yield in a collision (obtained by interpolation).

WOR(i=1-11, j=1-2, k=1-2, ℓ=1-5) ≡ normalization factors of yield for collision of incident particle of momentum index i and type j (see NI), outgoing particle distribution k (1 for ND=1,2 and 2 for ND=3,4) in problem material ℓ.

WSF ≡ test value to determine whether outgoing particle is a slow particle.

XMI(i=1-4) ≡ multiplicity or inelasticity of distribution D_i; also cumulative multiplicity or inelasticity.

YRA ≡ random number (of varying range).

CMES ≡ square of CME (GeV^2).

EEQL, PEQL ≡ energy, momentum of proton in lab with c.m. momentum equal to that of incident particle (GeV, GeV/c).

ELAB ≡ total energy of outgoing particle in lab (GeV).

EOCM ≡ total energy of outgoing particle in c.m. (GeV).

ETCM ≡ Lorentz factor η ($=\beta\gamma$) of c.m. in lab.

NPTR ≡ incident momentum bin corresponding to threshold momentum.

PCMX ≡ maximum momentum of outgoing particle in c.m. (GeV/c).

PICM ≡ momentum of incident particle in c.m. (GeV/c).

PLLO ≡ logarithm of lowest tabular momentum (0.1 GeV/c).

PTMX ≡ maximum value of PT (p_T) in GeV/c.

SQMO ≡ weight factor incurred in selecting distribution multiplied by the normalization factor of particle yields (for fast particles).

VMI1, VMI2, VMI3, VMI4 are auxiliary arrays connected to VMI by an equivalence statement enabling VMI to appear in a DATA statement.

WIPO(i=1-11, j=1-2, k=1-5) inelasticity of π° (above threshold). Indices as in EBT.

XSUM ≡ total multiplicity or inelasticity of fast particles above threshold.

YPTR ≡ value of Y corresponding to threshold momentum.

PZCHI ≡ maximum value of p_Z^{cm} (PZ), in GeV/c.

VNORM(i=1-11, j=1-2, i=1-2, m=1-4) ≡ normalization factors of fast particle yields for standard materials m (other indices as in WOR).

XV. FUNCTION VSL

VSL calculates the multiplicity or inelasticity of slow nucleons above threshold. In the prescription of Ranft and Routti[6] the factor describing the angular dependence is normalized to unity. Therefore the multiplicity above threshold energy (T_{tr}) is

$$m_{>} = \int_{T_{tr}}^{T_o'} \frac{dN}{dT}\, dT = \int_{T_{tr}}^{T_o'} \left\{ \frac{V_a}{\alpha} \frac{\exp(-T/\alpha)}{[1-\exp(-T_o'/\alpha)]} + \frac{V_b}{\beta} \cdot \frac{\exp(-T/\beta)}{[1-\exp(-T_o'/\beta)]} \right\} dT$$

$$= V_a \frac{[\exp(-T_{tr}/\alpha)-\exp(-T_o'/\alpha)]}{[1-\exp(-T_o'/\alpha)]} + V_b \frac{[\exp(-T_{tr}/\beta)-\exp(-T_o'/\beta)]}{[1-\exp(-T_o'/\beta)]} \quad \text{(XV.1)}$$

where $T_o' = \min(T_o, 1)$ GeV, with T_o being the kinetic energy (nucleons) or total energy (pions) of the incident particle (in GeV), and

$$V_a = .066\sqrt{A} \qquad (T_o \le 0.1)$$
$$= .06\sqrt{A}\,[1.1+2.3(1.+\log_{10} T_o)^2] \qquad (0.1 < T_o \le 5)$$
$$= .46\sqrt{A} \qquad (T_o > 5)$$

$$V_b = .0077\sqrt{A} \qquad (T_o \le 0.1)$$
$$= .007\sqrt{A}\,[1.1+2.3(1.+\log_{10} T_o)^2] \qquad (0.1 < T_o \le 5)$$
$$= .0535\sqrt{A} \qquad (T_o > 5)$$

The coefficients $\alpha(\equiv a_1^{-1})$ and $\beta(\equiv a_2^{-1})$, where a_1, a_2 are the notation used in SPCAL (SPSEL), see XVI, are given by

$$\alpha = (0.018 + 0.0017\, T_o)(1 - 0.001A) \qquad (T_o \le 5)$$
$$= 0.0265(1 - 0.001A) \qquad (T_o > 5)$$

and $\beta = (0.105 + 0.01\ T_o)(1 - 0.001A) \qquad (T_o \leq 5)$

$= 0.155(1 - 0.001A) \qquad (T_o > 5)$

The kinetic energy carried off by slow particles above threshold is also readily obtained in closed form

$$T_> = V_a\ \alpha\ \frac{[(1+T_{tr}/\alpha)\exp(-T_{tr}/\alpha)-(1+T'_o/\alpha)\exp(-T'_o/\alpha)]}{[1-\exp(-T'_o/\alpha)]}$$

$$+\ V_b\ \beta\ \frac{[(1+T_{tr}/\beta)\exp(-T_{tr}/\beta)-(1+T'_o/\beta)\exp(-T'_o/\beta)]}{[1-\exp(-T'_o/\beta)]}\ . \qquad \text{(XV.2)}$$

The differences between the above and the Ranft-Routti prescription[6] are small and are introduced solely to improve calculation efficiency e.g. protons and neutrons are chosen from the same distribution and this allows them to be tracked simultaneously in RECORD.

GLOSSARY OF ARRAY AND CONSTANT NAMES (VSL)

A ≡ atomic weight of material.
N indicates multiplicity (N=1) of slow nucleons or energy (in GeV) carried off by slow nucleons (N=2).
T ≡ kinetic energy of incident nucleon (GeV).

SA ≡ $\sqrt{A}$
VA, VB ≡ V_a, V_b (see XV.1).

ALF, BET ≡ α,β (see XV.1).
ALT ≡ auxiliary storage (a.s.).
TIN ≡ T (GeV).
TOM ≡ maximum kinetic energy of outgoing nucleons (GeV).
TTR ≡ kinetic energy of nucleons at threshold (GeV).

EAIN, EBIN, EATR, EBTR ≡ a.s.

XVI. SUBROUTINE SPCAL (SPSEL)

SPCAL initializes SPSEL which selects the angle and momentum of a slow particle and calculates the weight factor. As in the selection of a fast particle the selection procedure may be based on multiplicity or inelasticity. The decision to select a slow particle from among all types of particles is made in CALKI. The selection procedure used here is largely analytical, in contrast with the numerical techniques of CALKI (XIV) and FPCAL (XXIV).

The differential yields of slow particles are taken from the work of Ranft and Routti,[6] but slightly simplified. Some formulae are reproduced here for convenience. The particle spectrum is written in factorized form

$$(dN/dTd\theta) = f(A,T_o,T)g(A,T,\theta), \qquad \text{(XVI.1)}$$

where A is the atomic weight, T_o and T are the kinetic energy, respectively of the incident and outgoing particle (in GeV) and θ is the polar production angle.

The factor $f(A,T_o,T)$ is assumed to be

$$f(A,T_o,T) = m_> \, c[\exp(-a_1 T) + b \exp(-a_2 T)], \qquad \text{(XVI.2)}$$

where $m_>$ ≡ average multiplicity above threshold and

$$c = \left\{ [\exp(-a_1 T_{min}) - \exp(-a_1 T_{max})]/a_1 + b\,[\exp(-a_2 T_{min}) - \exp(-a_2 T_{max})]/a_2 \right\}^{-1} . \qquad \text{(XVI.3)}$$

T_{min} is the kinetic energy corresponding to threshold and T_{max} = min $(T_o,1)$ GeV. The coefficients a_1 and a_2 are given by

$$a_1 = [(0.018 + 0.0017\ T_o')(1. - 0.001A)]^{-1}, \qquad \text{(XVI.4)}$$

and

$$a_2 = [(0.105 + 0.01\ T_o')(1. - 0.001A)]^{-1}, \qquad \text{(XVI.5)}$$

with

$$T_o' \begin{cases} = 0.1 \ (T_o < 0.1 \text{ GeV}) \\ = T_o \ (0.1 \text{ GeV} \le T_o \le 5 \text{ GeV}) \\ = 5 \ (T_o > 5 \text{ GeV}). \end{cases}$$

The factor b in equation (XVI.2) is given by

$$b = (7a_2/60a_1)[1-\exp(-a_1\ T_{max})]/[1-\exp(-a_2\ T_{max})]. \qquad \text{(XVI.6)}$$

The angular dependence $g(A,T,\theta)$ is assumed to be:

$$\frac{dN}{d\Omega} = N(T)\exp(-\theta^2/\lambda) \qquad (0 \le \theta \le \pi/2)$$

$$= N(T)\exp(-\pi^2/4\lambda) \qquad (\pi/2 \le \theta \le \pi) \qquad \text{(XVI.7)}$$

where $\lambda = (0.12 + 0.00036A)T^{-1}$ and $N(T) = \left[\int g(A,T,\theta)\,d\Omega\right]^{-1}$.

A. Kinetic Energy Selection

1. Based on Particle Number

For this case the selection function can be taken to be directly proportional to the yield dN/dT, but normalized to unity, i.e.

$$S(T) = c[\exp(-a_1T) + b\ \exp(-a_2T)]. \qquad \text{(XVI.8)}$$

It is first decided whether to choose either from the first or from the second term, by comparing a random number with the integral (over the whole T region) of the first term, U_1,

$$U_1 = c[\exp(-a_1 T_{min}) - \exp(-a_1 T_{max})]/a_1, \qquad \text{(XVI.9)}$$

(a) if $r \leq U_1$, then the new selection function becomes

$$S_1(T) = c \exp(-a_1 T)/U_1. \qquad \text{(XVI.10)}$$

A random T is now to be selected from $S_1(T)$

$$r' = \int_{T_{min}}^{T} S_1(T)dT = [\exp(-a_1 T_{min}) - \exp(-a_1 T)] \qquad \text{(XVI.11)}$$

$$/[\exp(-a_1 T_{min}) - \exp(-a_1 T_{max})],$$

where r' is a random number on the range (0,1). Equivalently, since $r \leq U_1$ the ratio r/U_1 is likewise random number on the range (0,1). Therefore

$$r = c[\exp(-a_1 T_{min}) - \exp(-a_1 T)]/a_1, \qquad \text{(XVI.12)}$$

and

$$T = -\log [\exp(-a_1 T_{min}) - a_1 r/c]/a_1. \qquad \text{(XVI.13)}$$

It is easy to verify that the weight factor incurred in this selection procedure is the constant $m_>$.

(b) if $r \geq U_1$, then the particle is selected from the second term of equation (XVI.8)

$$S_2(T) = a_2 \exp(-a_2 T)/[\exp(-a_2 T_{min}) - \exp(-a_2 T_{max})]. \qquad \text{(XVI.14)}$$

But, if $r \geq U_1$, then

$$1 - r \leq U_2 = (cb/a_2)[\exp(-a_2 T_{min}) - \exp(-a_2 T_{max})], \qquad \text{(XVI.15)}$$

where U_2 is the integral of the second term over the whole T region and using analogous steps to those leading to (XVI.13)

$$T = -\log\,[\exp(-a_2\,T_{min}) - a_2(1-r)/cb]/a_2. \qquad \text{(XVI.16)}$$

Again the weight factor is $m_>$.

2. Based on Inelasticity

A selection function of the form

$$S(T) = kT\,\exp(-aT), \qquad \text{(XVI.17)}$$

leads to a transcendental equation T(r). To keep the selection function simple it is taken instead to be

$$S(T) = c'[\exp(-a_1'T) + b'\,\exp(-a_2'T)], \qquad \text{(XVI.18)}$$

where c' is a constant of proportionality and a_1', a_2' and b' are determined by

$$\begin{aligned} \exp(-a_i'\,T_{min}) &= k_i\,T_{min}\,\exp(-a_i'\,T_{min}) \\ \exp(-a_i'\,T_{max}) &= k_i\,T_{max}\,\exp(-a_i'\,T_{max}), \end{aligned} \qquad \text{(XVI.19)}$$

for i = 1,2 and

$$\begin{aligned} &\int_{T_{min}}^{T_{max}} b'\,\exp(-a_2'T)\,dT \Big/ \int_{T_{min}}^{T_{max}} \exp(-a_1'T)\,dT \\ &= \int_{T_{min}}^{T_{max}} bT\,\exp(-a_2T)\,dT \Big/ \int_{T_{min}}^{T_{max}} T\,\exp(-a_1T)\,dT. \end{aligned} \qquad \text{(XVI.20)}$$

These relations lead to

$$a_i' = a_i - \log(T_{max}/T_{min})/(T_{max} - T_{min}) \quad (i=1,2) \tag{XVI.21}$$

$$\begin{aligned} b' = {} & b(a_2' a_1^2)/(a_1' a_2^2) \\ & \cdot[(1+a_2T_{min})\exp(-a_2T_{min}) + (1+a_2T_{max})\exp(-a_2T_{max})] \\ & \cdot[\exp(-a_1'T_{min}) - \exp(-a_1'T_{max})] \\ & \cdot[(1+a_1T_{min})\exp(-a_1T_{min}) + (1+a_1T_{max})\exp(-a_1T_{max})]^{-1} \\ & \cdot[\exp(-a_2'T_{min}) - \exp(-a_2'T_{max})]^{-1}. \end{aligned} \tag{XVI.22}$$

Exactly the same selection procedure can be used now as for the selection based on multiplicity, using the new constants a_1', a_2', b' and c'. However, the selection function is no longer proportional to the assumed (dN/dT) and hence the weight factor is no longer constant.

(a) if $r \leq U_1'$, where

$$U_1' = c'[\exp(-a_1'T_{min}) - \exp(-a_1'T_{max})]/a_1' , \tag{XVI.23}$$

the selection is made as in equation (XVI.13) with the primed coefficients instead of the unprimed ones. The weight factor then becomes

$$W = (dN/dT)_1/[U_1' S_1'(T)], \tag{XVI.24}$$

where

$$(dN/dT)_1 = m_> c \exp(-a_1T), \tag{XVI.25}$$

and

$$S_1'(T) = a_1' \exp(-a_1'T)/[\exp(-a_1'T_{min}) - \exp(-a_1'T_{max})]. \tag{XVI.26}$$

Hence $$W = m_{>} c \exp(-a_1 T)/[c' \exp(-a_1' T)]. \qquad \text{(XVI.}$$

(b) if $r \geq U_1'$, the selection is made analogous to equation (XVI.14) and the weight factor is

$$W = m_{>} c\, b \exp(-a_2 T)/[c'b' \exp(-a_2' T)]. \qquad \text{(XVI.28)}$$

B. Selection of Polar Angle

The normalization constant N(T) from equation (XVI.7) is evaluated numerically (during initialization in SPCAL) for 101 equally spaced values ($0 \leq T \leq 1$ GeV).

The selection of θ follows the selection of T. The selection function is assumed to be

$$S(\theta) = A\, \theta \exp(-\theta^2/\lambda) \qquad (0 \leq \theta \leq \pi/2)$$

$$\text{and} \qquad S(\theta) = A(\pi/2)\sin\theta \cdot \exp(-\pi^2/4\lambda) \qquad (\pi/2 \leq \theta \leq \pi) \qquad \text{(XVI.29)}$$

$$\text{where} \quad A = \left\{ (\lambda/2)[1 - \exp(-\pi^2/4\lambda) + (\pi/2)\exp(-\pi^2/4\lambda) \right\}^{-1} . \qquad \text{(XVI.30)}$$

(a) if $r \leq (A\lambda/2)[1 - \exp(-\pi^2/4\lambda)]$,

then the angle is chosen from the first distribution i.e.

$$r' = [1 - \exp(-\theta^2/\lambda)]/[1 - \exp(-\pi^2/4\lambda)], \qquad \text{(XVI.31)}$$

where r' is on the range (0,1).

Equivalently, $$r = (A\lambda/2)[1 - \exp(-\theta^2/\lambda)], \qquad \text{(XVI.32)}$$

hence $$\theta = [-\lambda \log(1 - 2r/A\lambda)]^{1/2}, \qquad \text{(XVI.33)}$$

and $$W = N(T) A^{-1} \theta^{-1} \sin\theta. \qquad \text{(XVI.34)}$$

(b) if $r \geq (A\lambda/2)[1 - \exp(-\pi^2/4\lambda)]$, (XVI.)

then $$S(\theta) = \sin\theta \quad \pi/2 \leq \theta \leq \pi, \tag{XVI.36}$$

hence $$\theta = \cos^{-1}(r'), \tag{XVI.37}$$

where r' is a random number on the range (0,1),

and $$W = N(T)2A^{-1}\pi^{-1}. \tag{XVI.38}$$

The azimuthal angle is assumed to be uniformly distributed over the region $(0,2\pi)$. This angle is chosen in the calling program (MAIN or RECORD).

GLOSSARY OF ARRAY AND CONSTANT NAMES (SPCAL)

B ≡ b, see (XVI.6) for multiplicity selection or b', see (XVI.22) for inelasticity selection.
I, J, V ≡ auxiliary storage (a.s.).
N index corresponding to outgoing kinetic energy for angle selection.
T ≡ kinetic energy of outgoing particle (GeV).
W ≡ mass of the nucleon (GeV).

A1, A2 ≡ a_1, a_2, see (XVI.4 and 5 for multiplicity selection or a_1', a_2' see (XVI.21) for inelasticity selection.
BB ≡ b, see (XVI.6) for inelasticity selection.
CN(i=1-101, j=1-5) ≡ normalization constants in the angular distribution, see (XVI.7) corresponding to kinetic energy index i and material index j.
DA ≡ (a.s.).
DT ≡ increment of T between tabular entries of selection function of polar angle (GeV).
NM ≡ problem material index.
PI ≡ 3.1416.
TH ≡ integration variable θ used in computing CN (radians).
TV(i=1-101, j=1-5) is used to decide whether to choose angle from forward or backward distribution (indices as in CN).

AA1, AA2 ≡ a_1, a_2, see (XVI.6) for inelasticity selection.
ANG ≡ polar angle of outgoing particle (radians).
DTH ≡ increment of TH used in computing CN (radians).
ETR ≡ kinetic energy of nucleon at threshold (GeV).
MAT ≡ atomic weight of material in fixed point notation.
NPI denotes selection according to multiplicity (=1) or inelasticity (=2).
NQI ≡ incident particle type (1=p, 2=n, 3=π^+, 4=π^-).
PTR ≡ threshold momentum (GeV/c).
TIN ≡ T (GeV).

TM1, TM2, TN1, TN2, TR1, TR2, TS1, TS2 ≡ a.s.

VLM(i=1-101, j=1-5) ≡ λ parameter in angular distribution (XVI.7) (indices as in CN).

YRA ≡ random number on range (0,1).

AINV(i=1-101, j=1-5) ≡ A^{-1} (see XVI.30), indices as in CN.

AMAT ≡ MAT in floating point notation.

AMDA ≡ λT, see (XVI.7).

BAKW, FORW ≡ a.s.

CINV ≡ c^{-1}, see (XVI.3) for multiplicity selection or $(c')^{-1}$, see (XVI.18) for inelasticity selection.

POUT ≡ momentum of outgoing particle (GeV/c).

TEST is used to decide whether to choose from first or second term in (XVI.8) or (XVI.18) in kinetic energy selection.

TOMX ≡ maximum kinetic energy of outgoing particle (GeV).

TOUT ≡ kinetic energy of outgoing particle (GeV).

TTIN ≡ T'_o, see (XVI.4 and 5).

WFAC ≡ weight factor incurred in selecting kinetic energy and polar angle.

CCINV ≡ c^{-1}, see (XVI.3) for inelasticity selection.

XVII. SUBROUTINE HITOR (HITORM)

Subroutine HITOR is normally supplied by the user. HITOR contains or reads information regarding the number of materials and the material parameters. HITORM assigns the material index given the location. Dimensions of the shielding configuration are transmitted by COMMON from BINCAL. This division between the two subroutines is rather arbitrary.

The present version has four different cases which can be called by specifying the parameter NGEOM (see III). For all four cases the outer boundaries of the problem are those of a cylinder.

GLOSSARY OF ARRAY AND CONSTANT NAMES (HITOR)

I ≡ auxiliary storage (a.s.).
N ≡ material index.
X, Y, Z ≡ Cartesian coordinates of particle's location (cm).

AT(i=1-5), ZT(i=1-5) ≡ atomic weight and atomic number of problem material with index i.
NE ≡ number of different materials present in problem.
RR ≡ square of the radial coordinate (cm^2).

ECR(i) ≡ critial energy of material with index i (MeV).
RHO(i) ≡ density of material with index i (g/cm^3).
ZIN ≡ Z-coordinate of configuration at which beam begins to interact (cm).

NOUT ≡ index signifying location outside problem boundaries.
RLIM ≡ outer radius of configuration (cm).
RLIS ≡ $RLIM^2$ (cm^2).
TARL, TARR ≡ target length, radius (cm) for case 2.
TARS ≡ $TARR^2$ (cm^2) for case 2.
TARZ ≡ position of target face in configuration (cm) for case 2.
TUNL, TUNR ≡ tunnel length, radius (cm) for cases 2-4.
TUNS ≡ $TUNR^2$ (cm^2) for cases 2-4.
ZLIM ≡ length of entire configuration (cm).

ISCAT indicates elastic scattering option (0=no, 1=yes).
NGEOM ≡ index distinguishing type of problem (see III).

XVIII. SUBROUTINE HPCAL (HPLOC, HPVOL)

This program is normally supplied by the user. The present version supports the four different cases described in III. It should be used only if any of these cases will provide the necessary information. Any changes made in this program may necessitate corresponding changes in the program PSCAL.

HPCAL defines the volume bins in which the momenta of the interacting (recording) particles are to be analyzed. In the present version the backward region (depths less than where the beam begins to interact) is excluded from momentum analysis. The rest is divided into cylindrical regions. The radial bin limits are chosen independent of depth but they are dimensioned so as to make a depth dependence readily possible.

HPLOC determines the volume bin for momentum analysis and returns this information to PSPEK by means of the depth and radial indices NZ and NR. Index NZ = 5 has been used to indicate that no momentum analysis is required.

HPVOL calculates the volume of the bins given its indices for use in PSPRT. In the present version for the case NGEOM=2 the bin NZ = 1, NR = 1 refers to the volume of the entire target.

GLOSSARY OF ARRAY AND CONSTANT NAMES (HPCAL)

I, J, K ≡ auxiliary storage (a.s.).
R ≡ radial coordinate of particle's location (cm).
X, Y, Z ≡ Cartesian coordinates of particle's location (cm).

NR, NZ ≡ indices identifying volume bin for momentum analysis.

RPL(i=1-4, j=1-4), RPH(i=1-4, j=1-4) ≡ lower and upper radial limits of volume bin i (depth index), j (radial index) in cm.
VOL ≡ volume of bin (cm^3).
ZIN ≡ Z-coordinate at which beam begins to interact (cm).
ZPL(i=1-4), ZPH(i=1-4) ≡ lower and upper depth limit of volume bins with depth index i (cm).

RLIM ≡ outer radius of configuration (cm).
TARL, TARR ≡ target length, radius (cm) for case 2.
TULI ≡ radial extent of tunnel (cm) for cases 2-4.
TUNL, TUNR ≡ tunnel length, radius (cm) for cases 2-4.
ZLIM ≡ length of entire configuration (cm).

NGEOM ≡ index distinguishing type of problem (see III).

TARVOL ≡ volume of target (cm^3) for case 2.

XIX. SUBROUTINE EMSHW (EMSHWR)

EMSHW initializes and EMSHWR calculates the energy deposition due to the electromagnetic (e.m.) shower resulting from π° decay. This program along with FDP, is called only when IEDEP $\geq$ 1 (user option).

The algorithm used to describe the spatial dependence of the energy deposited by e.m. showers is based largely on an emperical prescription which has been described elsewhere.[4] This prescription has been slightly changed, as noted below.

Briefly, whenever a π^+ or π^- propagating particle is selected from distribution D_3 or D_4 (weight = $W_{\pi^\pm}$), it is assumed that a π° has been created with weight = (1/2) 0.833 $W_{\pi^\pm}$ (see XVI). This π° is then assumed to decay instantaneously into 2 γ's. Only one γ is selected and its weight is 0.833 $W_{\pi^\pm}$. The probability distribution of the energy of the γ in the lab can be readily shown to be [14]

$$P(\varepsilon_\gamma) = p_\pi^{-1} \text{ for } 0.5(\varepsilon_\pi - p_\pi) \leq \varepsilon_\gamma \leq 0.5(\varepsilon_\pi + p_\pi) \qquad \text{(XIX.1)}$$

$$= 0 \text{ elsewhere.}$$

The selection function for ε_γ is taken to be

$$S(\varepsilon_\gamma) = 2\ \varepsilon_\gamma/p_\pi\varepsilon_\pi \text{ for } 0.5(\varepsilon_\pi - p_\pi) \leq \varepsilon_\gamma \leq 0.5\ (\varepsilon_\pi + p_\pi) \qquad \text{(XIX.2)}$$

$$= 0 \text{ elsewhere,}$$

and the weight factor is therfore $\varepsilon_\pi/2\varepsilon_\gamma$.
The selection algorithm is then

$$r = \int_{\frac{1}{2}(\varepsilon_\pi - p_\pi)}^{\varepsilon_\gamma} 2\varepsilon_\gamma d\varepsilon_\gamma/p_\pi\varepsilon_\pi\ , \qquad \text{(XIX.3)}$$

where r is a random number. From which

$$\varepsilon_\gamma = [p_\pi \varepsilon_\pi \, r + 0.25(\varepsilon_\pi - p_\pi)^2]^{1/2}. \qquad \text{(XIX.4)}$$

Given ε_γ the angle is uniquely determined. The expression

$$\cos\theta = \varepsilon_\pi[1 - m_\pi^2/(2\varepsilon_\gamma \varepsilon_\pi)]/p_\pi, \qquad \text{(XIX.5)}$$

is a convenient form for computation ($\cos\theta$ is close to unity in many cases).

The γ is now transported a distance ℓ randomly chosen from distribution[14]

$$P(\ell) = (7/9X_o)\exp(-7\ell/9X_o), \qquad \text{(XIX.6)}$$

where it is assumed to produce a pair of electrons. The distribution of the energy of either electron is approximately[17]

$$P(\varepsilon_e) = (9/7)\varepsilon_\gamma^{-3}(4/3\ \varepsilon_e^2 - 4/3\ \varepsilon_\gamma\varepsilon_e + \varepsilon_\gamma^2) \quad 0 \le \varepsilon_e \le \varepsilon_\gamma. \qquad \text{(XIX.7)}$$

Using again a selection function of the type used for $\pi^\circ \to \gamma\gamma$, see (XIX.2),

$$S(\varepsilon_e) = 2\varepsilon_e/\varepsilon_\gamma^2 , \qquad \text{(XIX.8)}$$

the weight factor becomes

$$W(\varepsilon_e) = (9/7)(4/3\ \varepsilon_e^2 - 4/3\ \varepsilon_\gamma\varepsilon_e + \varepsilon_\gamma^2)/(2\varepsilon_\gamma\varepsilon_e). \qquad \text{(XIX.9)}$$

The angle of the electron, θ', is assumed to be distributed according to Ref. 17

$$P(\theta') = 2\alpha^2\theta'/(\theta'^2 + \alpha^2)^2, \qquad \text{(XIX.}$$

where α is determined by the condition that the average of θ', determined from (XIX.10), equals the average angle as given by Stearns[18]

$$\langle\theta'\rangle^{1/2} = f(\varepsilon_\gamma,\varepsilon_e,Z)(m_e/\varepsilon_\gamma)\log(\varepsilon_\gamma/m_e), \qquad \text{(XIX.11)}$$

where $f(\varepsilon_\gamma,\varepsilon_e,Z) \simeq \varepsilon_\gamma/2\varepsilon_e$, ignoring the small Z dependence. It then follows that

$$\alpha = (m_e/\pi\varepsilon_e)\log(\varepsilon_\gamma/m_e). \qquad \text{(XIX.12)}$$

The algorithm which determines θ' becomes

$$\theta' = (y - \alpha^2)^{1/2}, \qquad \text{(XIX.13)}$$

with $$y = \left\{\alpha^{-2} - r\left[\alpha^{-2} - (\alpha^2 + \pi^2)^{-1}\right]\right\}^{-1}. \qquad \text{(XIX.13)}$$

For most applications the angles θ,θ' as determined by (XIX.5 and 13) are negligibly small. However in the important case of determining the maximum energy deposited (e.g. to study target heating) these angles can influence the result. (This case is also likely to be sensitive to elastic scattering.)

From this point on the (weighted) energy of the electron is deposited according to an empirical prescription based on rough fits to experimental data as well as MC calculation of e.m. showers.

The average longitudinal behavior of an electron initiated shower in material of atomic number Z is described in terms of the variable

$$\lambda_\ell = 325(\log \varepsilon_e)/(\log Z)^{1.73} \text{ in g.cm}^{-2}, \qquad \text{(XIX.14)}$$

where ε_e is the electron energy in MeV.

When distances are expressed in units of λ_ℓ, the fractional longitudinal energy deposition becomes roughly independent of the depth Z or of ε_e at sufficiently large ε_e. The result is then a "universal" function $F(Z/\lambda_\ell)$ which can readily be stored in the computer as an array, the elements of which give the fractions of the energy deposited at depths Z_i (stored in another array). Fits of $F(Z/\lambda_\ell)$ to experiment can be found in Ref. 4. Subsequently, the prescription has been modified to include deviations encountered at low ($\lesssim$ 6 GeV) energies. This is conveniently accomplished by representing the average longitudinal behavior as $kF(Z'/\lambda_\ell)$ where $Z' = Z + 1.2/(\log \varepsilon_e)^{1/2}$, with ε_e in MeV [but $F(Z'/\lambda_\ell) = 0$ for $Z \leq 0$]; k is a normalization factor. This prescription is based on data of Cranell[19] for electron initiated showers on Pb at 200 MeV and on Cu, Sn and Pb at 900 MeV. The fit is shown in Fig. 2.

The average radial dependence of the energy deposited at depth Z is assumed to be

$$G(Z,r) = kF(Z/\lambda_\ell)(0.46\lambda_\ell/ZX_M)\exp(-0.46\lambda_\ell r/ZX_M), \qquad \text{(XIX.15)}$$

where $X_M \equiv X_o E_s/E_c$ is the Molière unit, X_o is the radiation length, E_s is taken to be 21 MeV and E_c is the critical energy of the material in MeV. G(Z,r) denotes the energy deposited in a ring of constant width at depth Z. A simple algorithm results from interpreting the ratio (r/Z) in (XIX.15) as a (small) polar angle θ". The distribution (XIX.15) is then reproduced by

$$G(Z,\theta") = kF(Z/\lambda_\ell)g(\theta")$$

$$g(\theta") = (0.46\lambda_\ell/X_M)\exp(-0.46\lambda_\ell\theta"/X_M). \qquad \text{(XIX.16)}$$

Therefore, if an angle θ" is chosen according to g(θ") and if the energy is deposited along a ray at that angle according to $F(Z/\lambda_\ell)$ the distribution (XIX.15) will be reproduced.

Note that the function $F(Z/\lambda_\ell)$ is assumed to vanish for $(Z/\lambda_\ell) > 1.3$. It is represented in better detail at small values of (Z/λ_ℓ) where the variation is largest.

GLOSSARY OF ARRAY AND CONSTANT NAMES (EMSHW)

I, V ≡ auxiliary storage (a.s.)
P ≡ momentum of π° (GeV/c).
X, Y, Z ≡ Cartesian coordinates of point where π° is produced (cm).

AT(i=1-5), ZT(i=1-5) ≡ atomic weight, number of problem material with index i.
DR ≡ steplength for transport of e.m. shower through vacuum (taken equal to steplength of recording particles) in cm.
NL, NM, NN ≡ material indices determined by HITROM.
TH ≡ polar angle (γ,π°), $(e^\pm,\gamma)$ or angle of representative shower particle, in radians.
WE ≡ weight of π°.
WU ≡ weight of γ.
XB, YB, ZB ≡ Cartesian coordinates of γ,$e^\pm$ and representative shower particle (cm).

AEL ≡ log of energy of e^+ or e^- belonging to pair (energy in MeV).
CSD ≡ uniformly distributed distance on the range $(0,0.01\lambda_\ell)$, in cm, λ_ℓ is given by XIX.7.
CSE ≡ first step in tracking representative shower particle, in cm.
DCX, DCY, DCZ ≡ direction cosines of π°.
DDX, DDY, DDZ ≡ direction cosines of pair electron.
DEX, DEY, DEZ ≡ direction cosines of representative shower particle.
DFX, DFY, DFZ ≡ direction cosines of γ.
DPR ≡ first step to test if pair production has occured, uniformly distributed over 1/5 radiation length (cm).
ECR(i=1-5) ≡ critical energy of problem material i (cm).
EEL ≡ energy of pair electron (GeV).
EPI ≡ total energy of π° (GeV).
MBG, NBG are used to locate bin of UCVE from which to start tracking the representative shower particle.

PHI ≡ (random) azimuthal angle (radians).
PPD ≡ distance to pair production in radiation lengths.
RHO(i=1-5) ≡ density of problem material type i (g/cm^3).
RSQ ≡ a.s.
VBG ≡ (Z' - Z), see text.
WPI ≡ pion mass (GeV).

COTH ≡ cosine of angle (γ,π°).
CSLG ≡ λ_ℓ see (XIX.7) in cm.
CSLH ≡ step in tracking representative shower particle (cm).
CSLP(i=1-5) is used in computing CSLG for material with index i.
EGAM ≡ total energy of γ (GeV).
NBIN(i=1,40) is used to find the normalizing factor for UCVE (when starting bin $\neq$ 1).
NFLG indicates whether to redetermine volume bin (=1) or store event in previous bin (=0), in BINS.
NOUT indicates location outside problem boundaries.
RALF(i=1-5) ≡ RALG(i=1-5)/5.
RALG(i=1-5)≡ radiation length of problem material, index i (cm).
UCVE(i=1-25)≡ function $F(Z/\lambda_\ell)$ giving the fraction of the weighted electron energy to be deposited at depth (Z/λ_ℓ) from point of origin of electron.
USTP(i=1-25) ≡ steplength (in units of λ_ℓ) of i^{th} step used in tracking representative shower particle.
USTQ(i=1-25) ≡ additional steplengths (in units of λ_ℓ) needed where USTP changes, to ensure uniform distribution within each step.
VMLG(i=1-5) ≡ Molière unit, see (XIX.15) of problem material with index i.
VNOM(i=1-13) ≡ normalization of UCVE for representative shower particle starting at bin i.
VNOR ≡ a.s.
WEEL≡ weighted energy of pair electron (GeV).
YINV ≡ a.s.

XX. SUBROUTINE FDP (ENTRY EDP)

This subroutine calculates the energy deposition of the hadronic cascade by mechanisms other than the electromagnetic showers initiated by π° decay (see EMSHW). FDP is the initialization part and EDP records the energy deposition of the cascade by following a representative particle. For most applications, propagating particles may also serve as recording particles in this case.

The mechanisms considered are:

(1) Ionization. Ionization losses of charged particles including losses from bremsstrahlung and direct pair production are derived from existing compilations.[15,16] The values stored in an array expressing dE/dx in MeV/g cm^{-2} as a function of momentum (tabulated at 11 logarithmically spaced momenta ranging from 0.1 GeV/c to 1000 GeV/c) for each particle type and for each of the four standard materials. During initialization the dE/dx values for the problem materials are obtained by interpolation and are converted to units of GeV per collision length (λ) of the material. This is convenient, because the other mechanisms (below) are proportional to the number of interactions and hence to the weight of the particle at the beginning of the step multiplied by $[1-\exp(-\Delta/\lambda)]$, where Δ is the steplength. It can easily be shown that the average ionization energy deposited in a step per unit weight equals $(dE/dx)\lambda[1-\exp(-\Delta/\lambda)]$. Hence the ionization energy can be recorded along with the energy due to other mechanisms by depositing an amount $(dE/dx)\lambda W$ in each step ($W \equiv$ weight).

(2) De-excitation of Struck Nuclei. The nuclear excitation energy as a function of incident energy is assumed that given by equation (II.4). Not all this energy is assumed to be deposited energy[4] and allowance is made for the fraction of excitation energy spent in removal of nucleons from the nucleus ($\sim$ 8 MeV per nucleon). The remainder is

$$E^{*\prime} = \frac{0.9 + 0.97\ \tau_o}{8.9 + 0.97\ \tau_o} E^* \ , \qquad \text{(XX.1)}$$

where E^* is the excitation energy and $\tau_o = (10E^*/A)^{1/2}$ is the nuclear temperature (in MeV) for E^* expressed in MeV and A is the atomic weight of the nucleus. This energy $E^{*\prime}$ is assumed to be equally distributed between charged particle and neutron evaporation. The charged particle fraction $0.5E^{*\prime}$ is deposited locally, one half of the neutron fraction $0.2E^{*\prime}$ is deposited isotropically one interaction length away, the remaining $0.25E^{*\prime}$ being ignored.

(3) Particles Below Threshold. The energy of particles emitted below threshold, E_{BT}, is treated like $E^{*\prime}$, i.e. $0.5E_{BT}$ is deposited locally and $0.25E_{BT}$ is deposited one interaction length away. When a proton or pion will fall below threshold during the next recording step the following procedure is used. First the particle is transported one half of a recording step in its direction of motion. From that point its kinetic energy (plus one half the rest energy for pions) is deposited in ten steps at a distance of one collision length similarly to the low energy neutrons. This rather arbitrary algorithm is introduced to smooth large fluctuations which could result if all this energy were to be deposited at one point. Note also that the rest energy of pions should not be deposited locally anyway.

To speed up computation the amount of energy to be deposited at each step is calculated only once for neutrons (being tracked) and is obtained by linear interpolation over the range for charged particles. This avoids converting range into momentum at each step when it is unnecessary (i.e. when ISCAT = 0).

As in CALKI (XIV) a slight adjustment is made to simulate the "kaon" component of $\pi^{\pm}$. As a crude approximation, the effects of the kaons, denoted by E, can represented by assuming

$$E(K^{\pm}) = (1/2)E(p) + (1/2)E(\pi^{\pm}) \qquad \text{(XX.2}$$

$$E(K^{\circ}) = E(n). \qquad \text{(XX.3)}$$

Then, using (XIV.4)

$$E_{comb}(\pi^{\pm}) = [1.05\ E(\pi^{\pm}) + 0.05\ E(p) + 0.1\ E(n)]/1.2\ , \qquad \text{(XX.4)}$$

where E_{comb} refers to the combined effect of (XIV.4). The factor 1.2 ensures normalization.

GLOSSARY OF ARRAY AND CONSTANT NAMES (FDP)

J, K, P ≡ auxiliary storage (a.s.).
N ≡ index of problem material.
R ≡ range of particle during tracking (cm).
U, V ≡ a.s.
W(i=1-3) ≡ mass of particle type i(1=p, 2=n, 3=$\pi^{\pm}$) in GeV.
X, Y, Z ≡ Cartesian coordinates of propagating particle (cm).

DR ≡ steplength used in tracking (cm), equal to steplength of recording particle.
EC(i=1-11, j=1-3, k=1-4) ≡ energy to be deposited locally for particle with momentum index i, particle type j (see W) and standard material type k (1=Be, 2=Al, 3=Cu, 4=Pb), in GeV.
FW ≡ normalization factor used in locating where energy due to evaporation and subthreshold neutrons or of ranged-out particle is to be deposited.
IJ, IK ≡ a.s.
MQ ≡ type of particle to be tracked (see W).
NG, NH, NI, NL ≡ material indices determined by HITORM.
NQ ≡ type of propagating particle (see W).
NZ(i=1-4)≡ atomic number of standard material type i (see EC).
PP ≡ momentum of propagating particle (GeVc/).
RA(i=1-11, j=1-3, ℓ=1-5) ≡ range in cm of particle type j (see W) with momentum index i in problem material type ℓ.
RR ≡ range of charged propagating particle (cm), at point of origin; for a neutron this is replaced by its momentum (GeV/c).
RT(i=1-4, j=1-5) ≡ range at threshold for particle type i(1=p, 2=n, 3=π^{+}, 4=π^{-}) in problem material type j, in cm.
VX, VY, VZ ≡ direction numbers along which energy due to evaporation and subthreshold neutrons or ranged out particle is to be deposited.

WE ≡ weight of propagating particle.

WF ≡ weighted number of interacting particles in first step.

XC, YC, ZC ≡ Cartesian coordinates of particle being tracked (cm).

XD, YD, ZD ≡ Cartesian coordinates of location where energy due to evaporation and subthreshold neutrons is to be deposited (cm).

XX ≡ a.s.

ZT(i=1-5) ≡ atomic number of problem material type i.

DBB ≡ first step (cm), randomly distributed over steplength DR according to $\lambda^{-1} \exp(-DBB/\lambda)$.

DCX, DCY, DCZ ≡ direction cosines of recording particle.

DDX, DDY, DDZ ≡ direction cosines of particle being tracked.

EAD ≡ energy carried off by evaporation and subthreshold neutrons or twice the energy of ranged-out particle (GeV). One half of this is deposited one interaction length away in a random direction.

EBD ≡ energy to be deposited locally (GeV).

EBT(i=1-11, j=1-2, k=1-5) ≡ fraction energy carried off by particles below threshold from collision of particle with momentum index i, type j (1 = n, 2 = π) in problem material type k.

EDM(i=1-5, j=1-2) ≡ average number of stars produced within a step of length DR in material type i by particle type j.

EDR(i=1-5, j=1-5, k=1-2) ≡ survival probability of particle (type k) after a step of length DR from material i to material j.

EKM ≡ kinetic energy (nucleons) or total energy (pions) of particle which will fall below threshold after the next step (GeV).

KAM is set equal to 10 when depositing energy from ranged-out charged particle (=1 for energy due to evaporation and subthreshold neutrons).

KAN ≡ flag (=1) when charged particle will fall below threshold in the next recording step (=0, otherwise).

NPT, VPT are used to find the energy to be deposited for neutrons being tracked.

PIP, TIP ≡ tabular momenta (GeV/c) and corresponding energy (kinetic-for nucleons, total-for pions) in GeV.

RHO(i=1-5) ≡ density of material type i (g/cm^3).

WAD ≡ amount of energy to be deposited (GeV) in BINS.

WCO ≡ adjusted weight cut off of calculation.

WRO ≡ weight cut off of calculation.

YRB ≡ random number on the range (0,1).

ZLN(i=1-5, j=1-2) ≡ collision length in material i, particle type j (see EBT) 1 cm.

DEDX(i=1-11, j=1-3, k=1-4) ≡ ionization energy loss ($MeV/g\ cm^{-2}$) for charged particles (indices as in EC).

EION(i=1-11, j=1-3, k=1-5) ≡ ionization energy to be deposited in recording step (GeV), indices as in RA.

ELOC(i=1-11, j=1-3, k=1-5) ≡ energy to be deposited locally in recording step (GeV), indices as in RA.

NFLG indicates whether to redetermine volume bin (=1) or store event in previous bin (=0), in BINS.

NOUT indicates location outside problem boundaries.

ISCAT indicates energy deposition option (0=no, 1=yes).

XXI. SUBROUTINE TRANS

This subroutine calculates the direction cosines of the produced particle given the polar and azimuthal production angle and the direction cosines of the incident particle. The method is the one described by Cashwell and Everett.[20] Since the procedure is applied many times in succession a precaution for renormalizing the new direction cosines is included. Since one often deals with very small angles, double precision arithmetic is used.

GLOSSARY OF ARRAY AND CONSTANT NAMES (TRANS)

A, B ≡ cosine, sine of polar production angles.
C, D ≡ cosine, sine of azimuthal production angle.
S, T, U ≡ direction cosines of produced particle.
X, Y, Z ≡ direction cosines of incident particle.

SZ ≡ auxiliary storage.
TH ≡ polar angle of production (radians).

PHI ≡ azimuthal angle of production (radians).
DEN ≡ normalization factor.

XXII. SUBROUTINE FPCAL (ENTRY FPSEL)

FPCAL initializes and FPSEL selects the polar angle and momentum of fast (i.e. selected from the HR production model) particles and calculates the weight factor incurred. Elsewhere in CALKI (CAMKI, SELKI) the distribution (D_1 - D_4, see III) and type of particle (p, n, π^+, π^-) have been determined.

The selection scheme is one suited for a general purpose calculation, i.e. without deliberate biasing. It is in principle not dependent on the particle production model and with minor modifications could be used for other production models.

The selection is performed differently according to whether the outgoing particle is a propagating or a recording particle. Briefly the propagating particles propagate the cascade and create stars. From these stars recording particles are generated for the purpose of estimating star density along their trajectory. A recording particle is tracked, uninterrupted by nuclear collision, until it (i) leaves the shield or (ii) weight drops below a preset limit or (iii) if the particle is charged, its momentum falls below the threshold momentum of the calculation.

For a recording particle an unbiased selection function is proportional to the differential yield

$$S_j(p,\Omega) = (dN/dpd\Omega)_j/\bar{M}_j, \qquad \text{(XXII.1)}$$

where the subscript j denotes the distribution, D_j, from which to select the particle; $\bar{M}_j$ is the average multiplicity of particles (above threshold) belonging to D_j and appears in equation (XXII.1) as a normalization factor. For all but the most simple expressions of the differential yield, equation (XXII.1) cannot be used in its exact form. Either simplified analytical approximations or numerical methods must be resorted to (see below).

For a propagating particle an efficient scheme is to sample proportionately to the average total number of stars produced in

all subsequent generations. Using the approximate rule that star production in a large block is roughly proportional to the energy of the particle, the selection function adopted is

$$S_j(p,\Omega) = (dN/dp\ d\Omega)_j\ E_j/\bar{I}_j\ , \qquad \text{(XXII.2)}$$

where E_j is taken as the (lab) kinetic energy for nucleons and total energy for pions; $\bar{I}_j$ is the inelasticity (above threshold) of particles belonging to distribution D_j.

The (lab) momenta and angles appearing in equations (XXII.1) and (XXII.2) and which are also the outputs to be generated by the selection routine are rather poorly suited variables for describing high energy processes. A better set of variables, adopted here, is

$$x = \exp\ (-kp_T^2), \qquad \text{(XXII.3)}$$

where p_T is the transverse momentum; normally $k \simeq 4\,(\text{GeV/c})^{-2}$, but this may be varied

$$y = p_Z^{cm}/p_{Z,max}^{cm}\ , \qquad \text{(XXII.4)}$$

where p_Z^{cm} and $p_{Z,max}^{cm}$ are the longitudinal momentum in the center of mass and its maximum value respectively. For the present purpose, there is no fundamental significance to this choice of variables. This advantage is that the production probabilities vary reasonable smoothly in terms of the variables x,y and the incident momentum, p_{inc}.

It is clear that the ranges of x and y are

$$\exp(-kp_{max}^{cm^2}) \leq x \leq 1, \qquad \text{(XXII.5)}$$

and

$$\begin{aligned} 0 \le y \le 1 \quad & \text{(for distribution } D_1\text{)} \\ -1 \le y \le 0 \quad & (D_2) \\ -1 \le y \le 1 \quad & (D_3, D_4). \end{aligned} \tag{XXII.6}$$

FPCAL prepares tabulated selection functions to select x, given p_{inc}; and also to select y, given p_{inc} and x. There are different sets of tables for recording particles and propagating particles, for each of the four distributions and for up to eleven logarithmic incident momentum bins ranging from 0.1 GeV/c to 1000 GeV/c. A considerable amount of both computation and storage space is required to prepare the tables for the selection scheme. To minimize this, the tables are not created separately for each material present in the calculation. Advantage is taken of the fact that the HR model predictions are not very sensitive to nuclear mass, and only one set of such tables is computed. In the case of a multimedium calculation the yields upon which the tables are based are calculated for a fictitious material with an atomic weight equal to the average atomic weight of all materials present. However it must be emphasized that when calculating the weight factor

$$W = S^{-1}(x,y)\,(dN/dxdy), \tag{XXII.7}$$

(dN/dxdy) is calculated for the actual material in which the collision takes place. The use of a single "average" material may thus affect the efficiency of the calculation but does not introduce systematic errors. Since the HR model is rather insensitive to nuclear mass it follows that the loss in efficiency is expected to be small.

The x selection tables contain the integral probabilities

$$P(x_i) = k \int_{x_{min}}^{x_i} dx' \int_{y_{min}}^{y_{max}} (dN/dx'dy)\, dy, \tag{XXII.8}$$

for recording particles and

$$P(x_i) = k \int_{x_{min}}^{x_i} dx' \int_{y_{min}}^{y_{max}} (dN/dx'dy)\, E_{lab}(x',y)\, dy, \qquad \text{(XXII.9)}$$

for propagating particles. This is convenient for sampling by direct comparison with a random number in the standard fashion. For each of the 11 momentum bins there is a table for $P(x)$ containing 11 entries equally spaced in the variable x. For convenience, $x_1 = x_{min}$ is adopted. Hence the first entry $P(x_1) = 0$ in every array.

To each of the entries in the selection tables of x corresponds a table containing 26 entries of the y selection probability

$$P(x_i,y_j) = k \int_{y_{min}}^{y_j} (dN/dx_i dy')\, dy' , \qquad \text{(XXII.10)}$$

for recording particles, and a similar table for propagating particles

$$P(x_i,y_j) = k \int_{y_{min}}^{y_j} (dN/dx_i dy')\, E_{lab}\,(x_i,y')\, dy' . \qquad \text{(XXII.11)}$$

Again the convention $y_1 = y_{min}$ is used which results in $P(x_i,y_1) = 0$.

In computing the selection tables some safeguards against undue fluctuations are introduced. Such fluctuations may arise from the approximate nature of the selection functions. Even in terms of the variables x and y the differential yield (or inelasticity) may vary rapidly in certain regions of phase space, particularly near the kinematical limits. Roughly, for this choice of variables, the variations tend to be largest where the yield, dN/dxdy, is relatively small. Two remedies can be invoked:

(i) one can oversample these infrequent events at the cost of a slight decrease in efficiency for sampling the rest of phase space or (ii) one can completely ignore particles produced below a certain probability level by making the selection function identically vanish. Both are used in FPCAL.

In calculating the entries $P(x_i)$ at each momentum, the difference between successive entries $P(x_{i+1}) - P(x_i)$ is always taken to be the larger of 0.01 and the actually calculated difference. (Incidentally, this ensures that particles having large p_T and hence generally large angles are well sampled.) In preparing the tables $P(x_i,y_i)$ the other approach is taken and whenever $[P(x_{i+1}) - P(x_i)]$ is calculated to be less than 0.001, it is set equal to zero. This is because typically a substantial number of the entries (particularly in the selection based on inelasticity) are either zero or very small ($<<0.001$) and oversampling could affect the overall efficiency of the calculation. Also, the particles thus excluded (generally of low energy in the lab) can be created easily in other ways (e.g. "slow" nucleons, or from incident particles at slightly different momenta or angles). This is likely to result in an underestimate of such particles but at insignificant levels compared with the overall accuracy of the calculation. A rough estimate of the frequency with which such events are ignored is obtained by assuming that all tabular entries are uniformly sampled. Given the four distributions, eleven different momenta and ten x-bins, the events occuring at the 0.001 level would be sampled on the average once per 440,000 tries. (This number is about twice as large as the "best statistics" run made with CASIM so far. Convergence is usually achieved in much less time depending on other factors such as the size of the shield.) It must be emphasized that the above criteria pertain only to a selection function used for general calculations. The user is adviced that the above mentioned techniques have not been investigated in great detail e.g. with respect to optimizing the limits on the difference between entries.

During computation the tables nearest in momentum to the momentum of the incident particle (on a logarithmic scale) are referenced. Since the calculation does not follow particles below a preset threshold momentum it is inefficient (though not necessarily incorrect) to select particles below this threshold. This is taken into consideration in the selection procedure.

First, for selecting pions from the distributions D_3 and D_4 it is determined whether it is indeed kinematically possible to create a pion above threshold. If not, no particle is created and the program returns with a weight of zero. (In the program CALKI, subprogram SELKI the various probabilities corresponding to the different distributions are adjusted for such threshold effects. However this is an approximate procedure and the probability for such "unphysical" pions may not identically vanish.)

Second, the momentum threshold may lower the upper bound on p_T (raise the lower bound on x). The absolute upper bound on p_T is

$$p_T^{max} = \left[\left(p_{max}^{cm} \right)^2 - |p_Z^{cm}|_{min}^2 \right]^{1/2} . \qquad \text{(XXII.12)}$$

Generally

$$p_Z^{cm} = \frac{M_p \varepsilon^{lab} - \sqrt{s}\, \varepsilon^{cm}/2}{p_i^{cm}} \qquad \text{(XXII.13)}$$

where M_p is the proton mass, ε^{lab} and ε^{cm} are the total energy of the outgoing particle in the lab and c.m. When $\varepsilon^{lab} = \varepsilon_{tr}^{lab}$ (total energy corresponding to threshold momentum), the smallest value of p_Z^{cm} occurs when $\varepsilon^{cm} = \varepsilon_{max}^{cm}$. Call this value $p_{Z,min}^{cm}$, then

$$|p_Z^{cm}|_{min} = \max\ (p_{Z,min}^{cm}, 0) , \qquad \text{(XXII.14)}$$

and p_T^{max} follows from equation (XXII.11). Therefore x must be chosen larger than

$$x_{min} = \exp\left(-k\ p_T^{max^2}\right). \qquad \text{(XXII.15)}$$

Third, given p_T the lower bound on y is established:

$$\left.\begin{aligned} y_{min} &= 0 \qquad (D_1) \\ &= -1 \qquad (D_2 - D_4) \end{aligned}\right\} \quad (p_T \geq p_{tr}), \qquad \text{(XXII.16)}$$

and

$$y_{min} = \frac{p_{z,min}^{lab}\ p_i^{lab} - (\varepsilon_i^{lab} - M_p)\ \varepsilon_{tr}^{lab}}{2\ p_i^{cm}\ p_{z,max}^{cm}} \qquad (p_T < p_{tr}), \qquad \text{(XXII.17)}$$

where $p_{z,min}^{lab} = (p_{tr}^{lab^2} - p_T^2)^{1/2}$.

For D_1, if $p_T < p_{tr}$, then y_{min} is taken to be the larger of the value of equation (XXII.16) and zero.

It is in principle possible that pions emitted with large backward momenta in the c.m. also travel backward in the lab. The maximum lab momentum approaches the value $M_p/2 - M_\pi^2/4M_p$ (or about 0.46 GeV/c). In case this exceeds p_{tr}^{lab}, one would have to select the pions from two nonadjacent regions in y. However the thermodynamic model predicts an extremely low yield for these particles and these backward pions (in the lab) are neglected.

Note that both the x and y selection tables span the full kinematical range of these variables, see (XXII.6). The limits on these variables due to threshold effects are introduced prior to each selection. The selection probabilities will therefore generally be larger than those in the table and the weights are to be adjusted accordingly.

The selection of x (between x_{min} and 1) is performed by comparing a random number r [between $P(x_{min})$ and 1] with the cumulative tabular entries $P(x_i)$ of the x selection array which pertains to the momentum nearest to the incident momentum. The value of $P(x_{min})$ is obtained by interpolation. When $p_T^{max} \neq p_{max}^{cm}$ [see equation (XXII.11)] the lowest permissible bin spans only a fraction of the range in x of the regular bins i.e. if the regular spacing is

$$\Delta x = (1 - x'_{min})/10. \qquad \text{(XXII.18)}$$

where $x'_{min} = \exp(-k\, p_{max}^{cm^2})$, then if

$$x_i < x_{min} \leq x_i + \Delta x, \qquad \text{(XXII.19)}$$

the lowest bin spans a range in x of $x_i + \Delta x - x_{min}$. It is possible that both the bin width and the difference between $P(x_i + \Delta x)$ and $P(x_{min})$ are then rather small numbers and that their quotient (which enters in the weight factor) becomes unreliable resulting in undue fluctuations. This is avoided by making the first comparison between $P(x_i + 2\Delta x)$ and $P(x_{min})$ on the interval $(x_{min}, x_i + 2\Delta x)$. For this lowest permissible bin the selection function of x within the bin has been assumed to be

$$S(x) = \left[(x_u^{1/2} - x_\ell^{1/2})\, x^{1/2} \right]^{-1}, \qquad \text{(XXII.20)}$$

where x_u, x_ℓ are the upper and lower level of the bin. This has been found empirically to represent the actual distribution fairly well. For all other bins the selection is made uniformly, i.e.

$$S(x) = 1/\Delta x\ . \qquad \text{(XXII.21)}$$

Given x, the y selection is performed from the table x_i nearest in x between $P(x_i, y_{min})$ and y_{max} (= 0 for D_2, = 1 D_1, D_3, D_4) in similar fashion to the x selection outlined above.

In this case the selection within the lowest permissible bin is also performed uniformly. However a sole exception to the uniform selection is made for the selection based on multiplicity for distributions D_3 and D_4 (pion production) near $y = 0$. Here the distribution is in some cases strongly peaked at $y = 0$. Therefore, within bin 14 which is centered at $y = 0$, the selection function is assumed to be

$$S(|y|) = a \exp(-a|y|)/[1-\exp(-a|y|)], \qquad \text{(XXII.22)}$$

and

$$S(y) = S(-y),$$

where a is determined by

$$(p_{14} - p_{13})/(p_{15} - p_{14}) = \exp(-0.08\ a), \qquad \text{(XXII.23)}$$

and p_n is the entry corresponding to the n^{th} bin. The constants a belonging to the different y selection arrays are evaluated in FPCAL.

The outgoing momentum and polar angle in the lab are derived from x and y. (A random azimuthal angle is assigned to the outgoing particle in MAIN or RECORD.) The weight factor is now calculated:

$$W = (dN/dp_T^2 dp_Z^{cm})\,(p_{Z,max}^{cm}/kx)\,(g_x g_y/f_x f_y), \qquad \text{(XXII.24)}$$

where the first factor is the differential yields as calculated by the program SPULAL and the second factor is de Jacobian of the transformation $\partial(p_T^2,\ p_Z^{cm})/\partial(x,y)$. The variables in the third factor are

$$g_x = \begin{cases} (x_u^{1/2} - x_\ell^{1/2})x^{1/2} & \text{lowest permissible bin} \\ \Delta x & \text{otherwise} \end{cases} \qquad \text{(XXII.26)}$$

$$g_y = \begin{cases} 2\exp(a|y|)[1-\exp(-a|y|)]/a & \text{for } |y| < 0.04 \ (D_3, D_4) \\ \Delta y & \text{otherwise} \end{cases} \qquad \text{(XXII.27)}$$

$$f_x = [P(x_i) - P(x_{i-1})]/[1 - P(x_{min})]$$

$$f_y = [P(x_i, y_j) - P(x_i, y_{j-1})]/[1 - P(x_i, y_{min})] \ . \qquad \text{(XXII.28)}$$

The symbols are defined above.

Finally, for a recording particle, it is determined if a particle with equal but opposite p_z^{cm} is above threshold in the lab. If so, the particle is split into two particles with equal weight (in RECORD). If not, the program returns a single particle and no splitting occurs in RECORD.

This procedure exploits the fact that these particles have the same production probability and hence with little effort the sampling efficiency is considerably increased. A few adjustments are necessary since the "reflection" of a particle may belong to a different distribution and type. These adjustments are made in RECORD.

GLOSSARY OF ARRAY AND CONSTANT NAMES (FPCAL)

J, K, L, N, V ≡ auxiliary storage (a.s.).

W(i=1-4) ≡ mass of particle type i (1=p, 2=n, 3=π^+, 4=π^-) in GeV.

X, Y ≡ integration variables x = $\exp(-kp_T^2)$ where k is the variable SLO(i=1-4) dependent upon particle type, y = $p_Z^{cm}/p_{Z,max}^{cm}$.

DC ≡ $dN/dxdy$ (unnormalized).

DX, DY ≡ (1) integration intervals of x,y (2) width of bin from which to select x,y.

EX ≡ a.s.

FX, FY ≡ probability of choosing x,y between two given bin limits, see (XXII.27).

GX ≡ a.s.

IX, IY ≡ index of upper limit of bin from which to select x,y.

JP ≡ index of tabular momentum nearest to momentum of incident particle (also a.s.).

JX ≡ index of lower limit of bin from which to select x (also a.s.).

KL, KM ≡ a.s.

KO ≡ index of type of particle (1=p, 2=n, 3=$\pi^\pm$).

MX, MY ≡ index of entry corresponding to $x_i + 2\Delta x$, where x_i satisfies $x_i \leq x_{min} \leq x_i + \Delta x$, see text following (XXII.19); ditto for MY.

NM ≡ material index (=1 for single material, =6 for multiple material) or (in FPSEL) index of problem material.

NO ≡ type of distribution (see III).

NQ, NV, NX, NY ≡ a.s.

PT ≡ transverse momentum (GeV/c).

PZ ≡ longitudinal momentum in cm (GeV/c).

SE(i=1-2) ≡ a.s.

VK(i=1-2, j=1-11, k=1-10) ≡ corresponds to a in (XXII.22); i is the type particle (1=π^+, 2=π^-), j = momentum index and k = x-bin of outgoing particle.

VX, VY, XS, XX ≡ a.s.

ANG(i=1-2) ≡ polar angle of outgoing particle(s) in radians.
ARG ≡ a.s.
CME ≡ total energy in c.m. of pp collision (GeV).
DCE ≡ E(dN/dxdy), where E = kinetic energy (nucleons), total energy (pions) in GeV.
DDY ≡ (constant) width of bin from which to select y i.e. prior to threshold corrections.
DEN ≡ a.s.
DPL ≡ (constant) difference $\log(p_i) - \log(p_{i+1})$, where p_i are the tabular momenta.
ETR(i=1-4) ≡ total energy corresponding to threshold momentum (GeV).
EVK(i=1-2, j=1-11, k=1-10) ≡ 1 - exp(-0.04a), where a is defined in (XXII.22), for indices see VK.
FLO ≡ smallest value of y kinematically allowed, including effects of threshold (see XXII.16 and 17).
GCM ≡ Lorentz factor γ of c.m. in lab.
JMN, JMX ≡ minimun and maximum tabular momentum index for which to evaluate arrays.
NQI ≡ type of incident particle.
NUO indicates the number of recording particles generated (=1 or 2).
PIN ≡ momentum of incident particle (GeV/c).
PTR ≡ threshold momentum (GeV/c).
PTS ≡ square of PT $(\text{GeV/c})^2$.
PZL ≡ longitudinal momentum in lab (GeV/c).
PZT ≡ $p^{cm}_{Z,min}$ (see XXII.14), in GeV/c.
QUO ≡ a.s.
SET(i=1-2, j=1-4, k=1-11, ℓ=1-11) ≡ x-selection array; i=1-2 refers to multiplicity or inelasticity selection, j = distribution of outgoing particle, k = momentum bin of incident particle, ℓ = x bin of outgoing particle.
SEZ(i=1-2, j=1-4, k=1-11, ℓ=1-10, m=1-26) ≡ y-selection array, indices as in SET and m = y-bin of outgoing particle.

SLO(i=1-4) ≡ coefficient k in $(GeV/c)^{-2}$ in $x = \exp(-kp_T^2)$;
i = distribution index.
XLO, YLO ≡ absolute lower limit of x,y.
YMI ≡ minimum (non zero) difference between successive entries in y selection arrays.
YRA ≡ random number on range (TMIN,1) or (SMIN,1).

CMES ≡ square of CME (GeV^2).
ECMX, PCMX ≡ maximum (total) energy, momentum of outgoing particle in c.m. (GeV).
EEQL, PEQL ≡ energy, momentum of proton in lab with c.m. momentum equal to that of incident particle (GeV, GeV/c).
ELAB ≡ total energy in lab of outgoing particle (GeV).
ELMX ≡ maximum (total) energy in lab of outgoing particle (GeV).
ETCM ≡ Lorentz factor $\eta(=\beta\gamma)$ of c.m. in lab.
PICM ≡ momentum of incident particle in c.m. (GeV).
PIHI, PILO ≡ logarithm of maximum and minimum of tabular momenta (1000 GeV/c and 0.1 GeV/c, resp.).
POUT(i=1-2) ≡ momentum in lab of outgoing particle(s) in GeV/c.
PTMX ≡ p_T^{max}, see (XXII.12), in GeV/c.
PTRS ≡ square of PTR $(GeV/c)^2$.
SETT, SETU, SEZT, SEZU ≡ a.s.
SMIN, TMIN ≡ (cumulative) probability corresponding to y_{min}, x_{min} of y or x selection array.
WFAC ≡ weight factor incurred in the selection of momentum and angle.
XMIN ≡ x_{min}, see (XXII.15).
XTRA, YTRA are extra weight factors when x,y selection within a bin is non-uniform, see (XXII.25 and 26).

PZCFL ≡ $y_{min}\, p_{Z,max}^{cm}$, see (XXII.17) in GeV/c.
PZCHI ≡ $p_{Z,max}^{cm}$, the maximum longitudinal momentum of outgoing particle in c.m. (GeV/c).
PZILLO ≡ $p_{Z,min}^{lab}$, see (XXII.17) in GeV/c.

XXIII. FUNCTION SPULAL, ENTRY SPULAM, SPUL

This program calculates production yields of p, π^+, π^- from p-nucleus collisions.

The basic algorithm for calculating HR particle yields consists of evaluating the sum of a few integrals (over the velocity parameter, λ, of the matter in the fireball). The present program is essentially a condensed and somewhat faster version of the relevant parts of Ranft's program SPUKJ.[5] The user should consult Ref. 5 for detailed information. For most applications it should be unnecessary to make changes in this subprogram and what follows may then be largely ignored.

SPULAL initializes the maximum number of integration steps (120). Also, some quantities (those independent of target material) are tabulated for later reference (in SPUL). These include N_B (partition function for one brought-in baryon) and T (temperature) as a function of energy density, ε, in the fireball. Since N_B and T vary most rapidly for small ε, two tables (each with 201 entries) are prepared: one for use in the range

$$1.2 \times 10^9 \leq \varepsilon \leq 1.0 \times 10^{10} \text{ MeV}^4,$$

and the second for use in the range

$$4.5 \times 10^8 \leq \varepsilon \leq 1.2 \times 10^9 \text{ MeV}^4.$$

Below $\varepsilon = 4.5 \times 10^8$ MeV^4, N_B and T are evaluated directly during execution of SPUL.

SPULAM initializes the constants which depend upon target material (index NM). These are obtained by linear interpolation (using the atomic weight as the dependent variable) from the corresponding constants for the four standard elements (index N) fitted by Ranft[8] (Be, Al, Cu, Pb). The relatively small variation of these constants over this large A-range justifies this procedure

SPUL calculates $dN/dp_T^2\ dp_Z^{cm}$ (apart from normalization) from the input variables NO (type of distribution of outgoing particle), NM (material index), CME (total energy in c.m.), CMES (square of CME) PZ (longitudinal c.m. momentum of outgoing particle), PT (transverse momentum of outgoing particle).

One difference with SPUKJ is the integration procedure. In SPUKJ 30 equally spaced lengths are used along with Simpson's rule of integration. To increase speed of computation, the rectangular rule is used and the number of steps is variable. The integration interval is roughly proportional to the longitudinal c.m. momentum of the outgoing particle (between 1/120 and 1/10 of the entire interval). Further, the integrand is negligibly small outside a single connected region of the integration variable. This is exploited by exiting the integration loop once the contribution to the integral at a point is less than a preset fraction of the accumulated total.

The flow of control is indicated in Fig.3. The extra normalization present in CASIM (see above) is introduced in CALKI (entry SELKI).

GLOSSARY OF ARRAY AND CONSTANT NAMES (SPULAL)

I, J, K, N ≡ auxiliary storage (a.s.).
T ≡ temperature (GeV).

AG ≡ a.s.
BE ≡ Lorentz factor β corresponding to integration variable XY.
CB(i=1-4, j=1-4), CD(i=1-4, j=1-4) ≡ constants in kinematical cut-off function resp. for isobar decay and direct production; i = index of distribution see (III), j identifies standard element (1=Be, 2=Al, 3=Cu, 4=Pb).
CC ≡ normalization factor of direct production contribution (for distributions 3 and 4).
DK ≡ a.s.
EE ≡ energy of particle in fireball element restframe (GeV).
FL(i=1-120, j=1-6, k=1-6) ≡ velocity weight functions corresponding to integration variable XY(i), index j = 1 for through going particle, j = 3,4 for π^+,π^- from isobar decay; j = 5,6 for direct π^+, π^- production; index k refers to problem material.
GG ≡ Lorentz factor γ corresponding to integration variable XY.
H1(i=1-4, j=1-4), H2(k=1-3, i=1-4, j=1-4) ≡ constants in the velocity weight function (FL); i = index of distribution (see III), j = index of standard element, k distinguishes three different constants.
IQ ≡ a.s.
KD, KU ≡ flags which signal when integration over XY is complete.
NB ≡ index of "representative" isobar.
NM ≡ index of problem material.
NO ≡ index of distribution (see III).
NX, NY ≡ increment and starting point of integration loop (over XY).
PB(i=1-4, j=1-3) ≡ a.s. arrays used in calculating different terms (identified by i and j) contributing to the isobar decay mode of production.
PT ≡ transverse momentum of outgoing particle (GeV/c).

PZ ≡ longitudinal momentum of outgoing particle in c.m. (GeV/c).

T1(i=1-201),T2(i=1-201) ≡ pre-tabulated array representing temperature (GeV) as a function of energy density (in MeV^4).

WB(i=1-3) ≡ mass of representative isobar of index i (GeV).

WS(i=1-2) ≡ square of mass of nucleon (i=1), pion (i=2).

XX ≡ a.s.

XY(i=1-120) ≡ integration variable $|\gamma-1|/|\gamma_0-1|$, where γ and γ_0 are the Lorentz factors of the fireball element and incident proton in c.m.

ASE(i=1-4) ≡ atomic weight of standard materials (see CB).

BPM(i=1-3, j=1-4, k=1-6), BPS(i=1-3, j=1-4, ℓ=1-4) ≡ isobar weighting coefficients for problem and standard materials, resp.; i = index of representative isobar; j = index of distribution (see III); k = index of problem material; ℓ = index of standard material (see CB).

CMB(i=1-4, j=1-6), CMD(i=1-4, j=1-6) ≡ constants in kinematical cut-off function respectively for isobar decay and direct production; i = index of distribution (see III), j = index of problem material.

CME ≡ total energy in c.m. of pp collision (GeV).

DEL ≡ smallest integration interval of variable XY.

DEN(i=1-6, j=1-6) ≡ normalization factors of the velocity weight functions (indices i,j correspond to j,k to FL).

EDV(j=1-3, k=1-2) ≡ total energy of decay product in isobar rest-frame [j=index of representative isobar, k=1(p), 2(π)], in GeV.

EMX ≡ maximum energy of pion in fireball element (GeV).

GM1 ≡ γ_0 - 1, where γ_0 is the Lorentz factor of the incident particle in c.m.

HH1(j=1-4), HH2(i=1-4) ≡ a.s. arrays which correspond to H1, H2 and are used to evaluate FL for problem material NM (in SPULAM).

IDX ≡ a.s.

MAT ≡ atomic weight of problem material in fixed point notation.

PDI, PIB ≡ contribution to pion yield from direct, isobar production mode (up to normalization).

PDJ ≡ a.s.

PDV(j=1-3) ≡ momentum of decay products (p and π) in isobar restframe (GeV/c), j = index of representative isobar.

PIC ≡ a.s.

PIE ≡ 3.1416.

PML ≡ maximum longitudinal momentum of outgoing particle in c.m. corresponding to integration variable XY (GeV/c).

PMX ≡ maximum momentum of pion in fireball element (GeV/c).

PRB, PRC, PRD, PRT, PRU ≡ a.s.

PTS ≡ square of PT $(GeV/c)^2$.

VPA(i=1-4, j=1-4), VPB(i=1-4, j=1-4),WPA(i=1-4, j=1-4), WPB(i=1-4, j=1-4) ≡ constants in the normalization of direct π production (i=3 for π^+, i=4 for π^-) for standard element of index j (see CB).

VPM(i=1-4, j=1-6), VPN(i=1-4, j=1-6), WPM(i=1-4, j=1-6), WPN(i=1-4, j=1-6) correspond to VPA, VPB, WPA, WPB for problem material with index j.

CMES ≡ square of CME (GeV^2).

DELS, DELT ≡ integration intervals (of EDNS, EDNT) used in calculating the tables of T1, T2 (MeV^4).

EDNS, EDNT ≡ integration variables representing energy density in the fireball element for calculating the tables of T1, T2 (MeV^4).

EOCM(i=1-2) ≡ energy of outgoing particle in c.m. (1=nucleon, 2=pion) in GeV.

NINT ≡ maximum number of integration steps over XY to calculate thermodynamical yields.

PFIR(i=1-2) ≡ momentum of proton in fireball (GeV/c); i-1,2 indicate forward, backward moving fireball in c.m.

PIBB ≡ a.s.

SPUL ≡ $dN/dp_T^2dp_Z$ in c.m. (particles/GeV^3).

VMAT ≡ MAT in floating point notation.

VMMX ≡ maximum mass in fireball element (GeV).

VNBV ≡ values of N_B (see text) evaluated at XY(i).

VNB1(i=1-201), VNB2(i=1-202) ≡ tabular values of N_B versus energy density.

XXIV. FUNCTION TOTO

This function calculates the quantity TOTO (=temperature in GeV/0.16) as a function of the energy density (in MeV^4) and is identical with the similar subprogram of SPUKJ[5].

GLOSSARY OF ARRAY AND CONSTANT NAMES (TOTO)

C(i=1-45) ≡ interval between tabulated values of TT(i).
E ≡ energy density (MeV^4).
I ≡ auxiliary storage.

EE(i=1-45) ≡ tabular entries of energy density (in MeV^4) corresponding to TT(i).
TT(i=1-45) ≡ tabular entries of [temperature (GeV)/0.16].

TOTO ≡ quantity [temperature (GeV)/0.16] corresponding to E.

XXIV. FUNCTION EXIN

This program calculates the exponential integral $E_1(X) = \int_X^\infty [\exp(-t)/t]dt$ according to standard numerical methods.[21]

GLOSSARY OF ARRAY AND CONSTANT NAMES (EXIN)

X ≡ argument of the exponential integral.

EXIN ≡ exponential integral $E_1(X)$.

XXVI. EXAMPLE OF A BIASED PROBLEM

The star (or energy) densities and momentum spectra normally calculated by CASIM are not always the type of information best suited to the user. As an example, the application of CASIM to a highly specific problem is included. The problem was actually posed by T.Jenkins of SLAC.

Figure 4 shows the configuration. The object is to obtain the spectra of π^+ and of π^- at the two "detectors" in the location shown. Each detector is assumed to be 1 cm^2 in area placed perpendicular to the beam. Since no other information is required CASIM is to be altered to run in a dedicated manner for this problem.

The solution shown here follows the cascade throughout the configuration, pausing at every interaction of a propagating particle to generate a π^+ and a π^- at each of 9 momenta (20-180 GeV/c in steps of 20 GeV/c) in the direction of the detector.

Except for a few other minor changes this involves (1) writing the subroutine HITOR to correspond to the geometry (2) writing a special subroutine (COCAL) to generate the spectra in the manner described above (and print them).

A listing of the changes is included.

(1) HITOR should be readily understandable.

(2) In MAIN:

(a) The coordinates of the propagating particle are now expressed in double precision (since y is always a very large number and any Δy is generally very small).

(b) Reference to BINCAL has been omitted. The relevant information is all in HITOR.

(c) For each element (Al and Fe) COCAL performs the initialization of the subroutine for calculating the π spectra.

(d) The constant 22035 is added to the y coordinate of the beam determined in the usual fashion.

(e) The recording of primary particles is bypassed.

(f) Subroutine RECORD is replaced by COUNT which generates the π spectra.

(g) PRINT which is called after the MC has been completed, normalizes and prints out the spectra.

In BINCAL (entry BINPT) the program is re-routed to print out only the energy balance, total star count and stars as a function of generation number.

In CALKI (entry CAMKI) the integration over the HR spectrum is carried out to greater precision in view of the high threshold (20 GeV/c). Note that this has been introduced as a precaution and has not been tested with the coarser integration normally used.

In the program COCAL (entry COUNT) the spectra of π^+, π^- at the detector are calculated separately for the second generation and for the higher generations. First, certain kinematical quantities needed to calculate the production cross section are obtained. Next, the program determines for each detector (a) the distance from the point of production (b) the polar angle with respect to the direction of the propagating particle (c) the direction cosines of the π^+ and π^- members of the spectra. Then by stepping through the configuration (d) the momentum loss and (e) the weight reduction due to absorption are determined. (The steplength, here taken as 20 cm is easily varied.) Finally the production cross section is evaluated. Recalling that the program SPUL yields $dN/dp_Z^C dp_T^2$ this is multiplied by the Jacobian

$$J = \varepsilon^C \, p^{\ell 2}/\pi \, \varepsilon^{\ell}, \qquad \text{(XXVI.1)}$$

to yield $dN/dp\, d\Omega$. An estimate of the contribution due to n^{th} generation of propagating particles expressed as particles (π^+ or π^-)/(GeV/c) (unit area) (steradian) (inc. p) is then

$$W_n \, A(dN/dp\, d\Omega)(\Delta\Omega/\Delta A)/\Delta\Omega' \, , \qquad \text{(XXVI.2)}$$

where W_n is the weight of the propagating particle. The absorption factor $A = \prod \exp(-\ell_i/\lambda_i)$ with ℓ_i, λ_i being the pathlength and collision length of the particle in material i; $\Delta\Omega/\Delta A$ may be taken as $\cos \theta_Z/r^2$ with r being the distance to the detector and θ_Z the angle with the beam direction. Finally the angles θ_Z are binned to obtain the angular distribution at the detector and $\Delta\Omega'$ represents the solid angle of the bin. Note that the approximation for $\Delta\Omega/\Delta A$ holds only when the detector is always sufficiently far from the point of production otherwise this must be determined more accurately.

Since the interest is mainly in the first few generations (because of the high threshold) and since typical lengths are much longer than λ, a rather large collision length multiplier is used.

A glossary of the subroutine COCAL follows.

GLOSSARY OF ARRAY AND CONSTANT NAMES (COCAL)

I, V ≡ auxiliary storage (a.s.).

M distinguishes pions from the interactions of primaries (=1) and higher generations (=2).

N ≡ material index of location of collision of propagating particle.

W(i=1-4) ≡ mass of particle (1=p, 2=n, 3=π^+, 4=π^-) in GeV.

X, Y, Z ≡ Cartesian coordinates of location of collision of propagating particle (cm).

EC ≡ total energy of outgoing particle in c.m. (GeV).

EL ≡ distance from collision site to π-counter (cm).

IA, IC, IP, IQ, MM ≡ a.s.

NA ≡ material index (1=Al, 2=Fe).

PO(i=1-9) ≡ discrete momenta at which to evaluate π^+,π^- yield at counters (GeV/c).

PT ≡ transverse momentum of outgoing particle (GeV/c).

PZ ≡ longitudinal momentum of outgoing particle in c.m. (GeV/c).

WS(i=1-4) ≡ square of mass of outgoing particle (GeV^2).

XA, YA, ZA ≡ Cartesian coordinates of outgoing particle, while tracking (cm).

ANG ≡ polar angle of outgoing with respect to incident particle (radians).

CAL(i=1-2, j=1-9) ≡ (dN/dp dΩ) for π^+(i=1), π^-(i=2) and momentum PO(j) in particles/(GeV)·(ster) printed for debugging check.

CME ≡ total energy in c.m. of pp collision (GeV).

COL ≡ collision length of material NA, calculated in MAIN (cm).

CTR(i=1-2, j=1-2, k=1-6, ℓ=1-9, m=1-3) ≡ yield of particle type j and momentum ℓ (see CAL) in angular region k[1=(0-0.5), 2=(0.5-1), 3=(1-5), 4=(5-10), 5=(10-50), 6=(50-1571) in mrad] for counter i (see Fig. 4). The index m differentiates the yields due to primaries (=1), higher generations (=2) and

total (=3). Units are particles/[(cm^2)·(ster)·(GeV/c)·(inc. proton)].

DCX, DCY, DCZ ≡ direction cosines of propagating (incident) particle.

DDX, DDX, DDY ≡ direction cosines of outgoing particle (aimed at a counter).

NQI ≡ type of propagating particle (see W).

PFS(i=1-2) ≡ [1-exp(-Δ/λ_i)], where Δ is the step length used in tracking and λ_i = collision length in material i (see NA).

PIN ≡ momentum of propagating particle (GeV/c).

PZL ≡ longitudinal momentum of outgoing particle in lab (GeV/c).

SNA ≡ sin (ANG).

THL(i=1-7) ≡ limits of angular regions (see CTR) in which particles at detector are analyzed, in radians.

XCT(i=1-2), YCT(i=1-2), ZCT(i=1-2) ≡ Cartesian coordinates of location of counter i (cm).

ABAR ≡ distance traversed through materials in units of collision lengths.

ABSF ≡ weighting factor modifying pion yield at counter due to absorption, distance from collision and angle of incidence (cm^{-2}).

ASTP(i=1-2) ≡ Δ/λ_i, see PFS.

CMEP ≡ square of total c.m. energy in collision of incident particle with a proton (GeV^2).

CMES ≡ square of CME (GeV^2).

COLT(i=1-2, j=1-2) ≡ collision length in material i (see NA), particle type j(1=n, 2=π) in cm.

DPDX(i=1-2) ≡ momentum loss [(GeV/c)/cm] in material i (see NA). During initialization DPDX becomes the momentum loss in GeV/c per step Δ in tracking.

EEQL, PEQL, TEQL ≡ total energy, momentum, kinetic energy of a proton in lab with c.m. momentum equal to that of incident particle (GeV, GeV/c).

EOUT ≡ total energy of outgoing particle in lab (GeV).

FSTP ≡ first step in tracking outgoing particle, determined on the interval Δ according to $\exp(-x/\lambda_i)$, in c.m.

NUGN ≡ generation number of propagating particle.

NUIM (MAXST) ≡ current total of incident particles (and maximum number of stars to be calculated) used to trigger a WRITE statement of a few quantities useful in debugging.

PLOS ≡ total momentum loss of particle between origin and counter (GeV/c).

POUT ≡ momentum of outgoing particle (GeV/c) at origin.

STEP ≡ steplength Δ used in tracking particle (cm) to determine PLOS and ABSF.

TFAC ≡ a.s.

VJAC ≡ Jacobian, J (see XXVI.1).

WGHT ≡ weight of the propagating particle.

MAXST ≡ maximum number of unweighted particles to be generated during calculation.

SOLAN ≡ solid angle subtended by a counter of 1 cm^2 angular region (up to the factor R^{-2} which has been absorbed in ABSF).

REFERENCES

[1] J. Ranft, Particle Accelerators 3, 77 (1971).

[2] T. W. Armstrong et al., Nucl. Sci. Eng., 49, 82 (1973).

[3] A. Van Ginneken, "Weighted Monte Carlo Calculations of Hadronic Cascades in Thick Targets," NAL-FN-250 (1972).

[4] A. Van Ginneken and M. Awschalom, "High-Energy Particle Interactions in Thick Targets. I. Hadron Cascades, Shielding and Energy Deposition", Fermilab, Batavia, Illinois (1974).

[5] R. Hagedorn, Suppl. Nuovo Cim. 3, 147 (1965); R. Hagedorn and J. Ranft, Suppl. Nuovo Cim. 6, 169 (1968); H. Grote, R. Hagedorn and J. Ranft, "Atlas of Particle Spectra," CERN, Geneva (1970).

[6] J. Ranft and J. T. Routti, "Hadron Cascade Calculations of Angular Distribution of Secondary Particle Fluxes from External Targets and Description of Program FLUKU," CERN-II-RA/72-8 (1972).

[7] H. W. Bertini, Phys. Rev. 188, 1711 (1969).

[8] J. Ranft, "Secondary Particle Spectra According to the Thermodynamical Model. A Fit to Data Measured in p-Nucleus Collisions," TUL-36, Karl Marx Univ., DDR (1970).

[9] J. V. Allaby, et al., "High Energy Particle Spectra from Proton Interactions at 19.2 GeV/c," CERN-70-12 (1970).

[10] A. Van Ginneken, "Comparison of Some Recent Data on p-Nucleus Interactions with the Hagedorn-Ranft Model Predictions," Fermilab-FN-260 (1974).

[11] G. H. Trilling, "Pion and Proton Fluxes from High Energy Proton Collisions," UCID-10148, LRL (1966).

[12] J. Ranft and T. Borak, "Improved Nucleon-Meson Cascade Calculations," NAL-FN-193 (1969).

[13] G. Belletini et al., Nucl. Phys. 79, 609 (1966).

[14] B. Rossi, High Energy Particles, p. 81, Prentice Hall, Englewood Cliffs, N. J. (1952).

[15] W. H. Barkas and M. J. Berger, "Tables of Energy Losses and Ranges of Heavy Charged Particles," in Studies in Penetration in Charged Particles in Matter, NSS-39 NAD-NRC Pub. 1133, p. 103 (1964).

[16]D. Theriot, "Muon dE/dx and Range Tables: Preliminary Results for Shielding Materials," NAL-TM-229 (1970).

[17]W. Heitler, The Quantum Theory of Radiation, 3rd Ed., p. 259, Oxford Univ. Press, London (1954).

[18]M. Stearns, Phys. Rev. 76, 836 (1949).

[19]C. J. Cranell, Phys. Rev. 161, 310 (1967).

[20]E. D. Cashwell and C. J. Everett, Monte Carlo Methods for Random Walk Problems, p. 103, Pergamon Press, New York (1959).

[21]M. Abramowitz and I. Stegun, Handbook of Mathematical Functions, p. 231, N.B.S., Washington, D. C. (1964).

FIGURE CAPTIONS

Fig. 1. Abbreviated flow diagram of MAIN.

Fig. 2. Fraction of total energy deposited per unit λ_ℓ versus depth (in units of λ_ℓ for data of Cranell.[19]
■ 200 MeV e^- incident on Pb.
⊙, ◇, ⊡ 900 MeV e^- incident on Cu, Sn, and Pb.
——, - - -, ▬▬ show the fits (of Chapter XIX) at 200 MeV, 900 MeV and high energy (≳ 6 GeV) respectively.

Fig. 3. Flow of control for subprogram SPUL.

Fig. 4. Example of a biased problem. A beam of 0.24 x 0.24 cm is incident on the aluminum wall of a beampipe surrounded by iron. The spectra of π^+ and π^- at two "detectors" are to be calculated (to determine shielding requirements for decay muons).

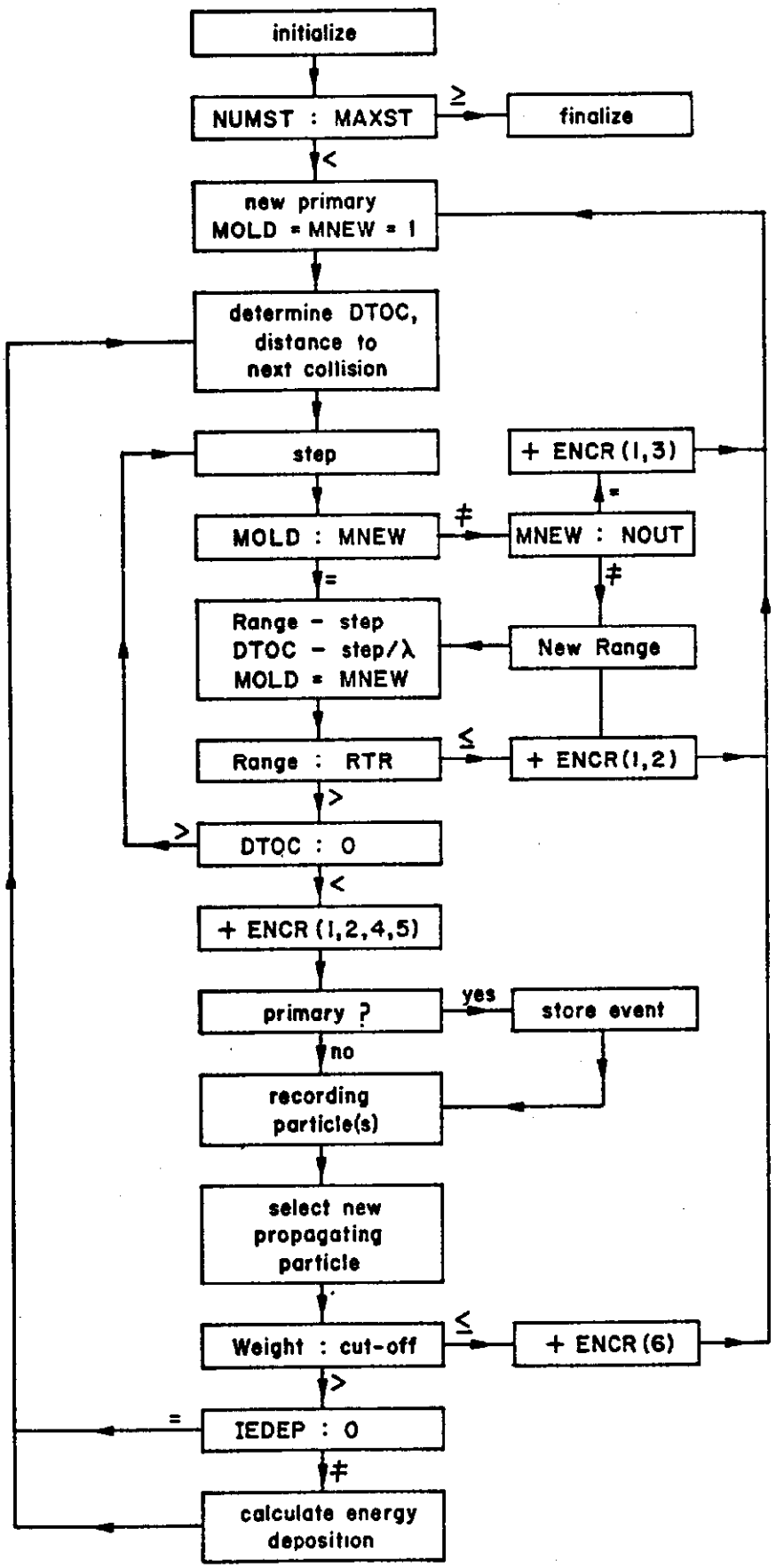

initialize
NUMST : MAXST
≥
finalize
<
new primary
MOLD = MNEW = 1
determine DTOC,
distance to
next collision
step
+ ENCR (1,3)
=
MOLD : MNEW
≠
MNEW : NOUT
=
≠
Range - step
DTOC - step/λ
MOLD = MNEW
New Range
Range : RTR
≤
+ ENCR(1,2)
>
>
DTOC : 0
<
+ ENCR (1,2,4,5)
primary ?
yes
store event
no
recording
particle(s)
select new
propagating
particle
Weight : cut-off
≤
+ ENCR (6)
>
=
IEDEP : 0
≠
calculate energy
deposition

Fig. 1

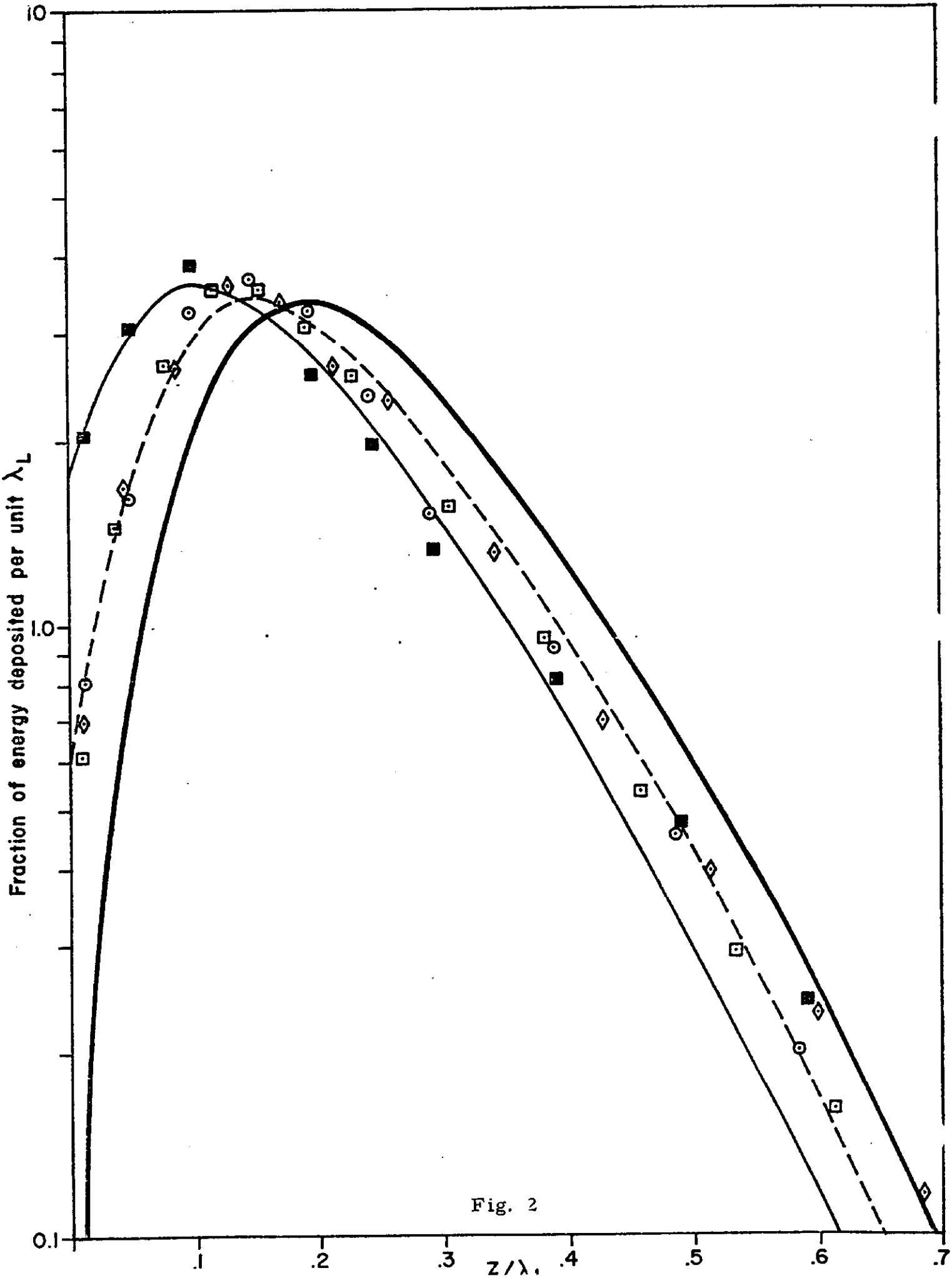

10
1.0
0.1
Fraction of energy deposited per unit λ_L
.1
.2
.3
.4
.5
.6
.7
z/λ_L

Fig. 2

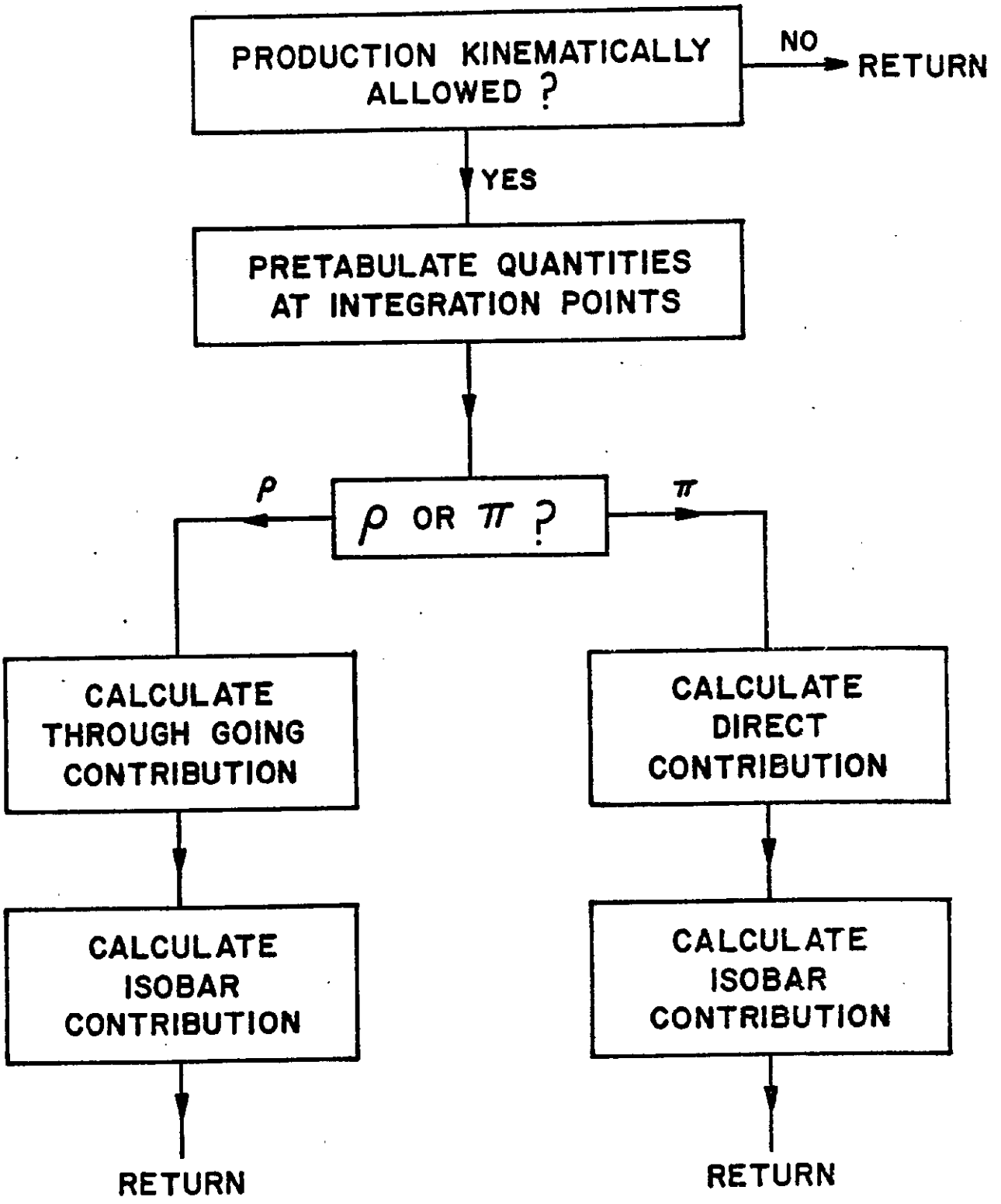

PRODUCTION KINEMATICALLY ALLOWED ?
NO
RETURN
YES
PRETABULATE QUANTITIES AT INTEGRATION POINTS
ρ
ρ OR π ?
π
CALCULATE THROUGH GOING CONTRIBUTION
CALCULATE DIRECT CONTRIBUTION
CALCULATE ISOBAR CONTRIBUTION
CALCULATE ISOBAR CONTRIBUTION
RETURN
RETURN

Fig. 3

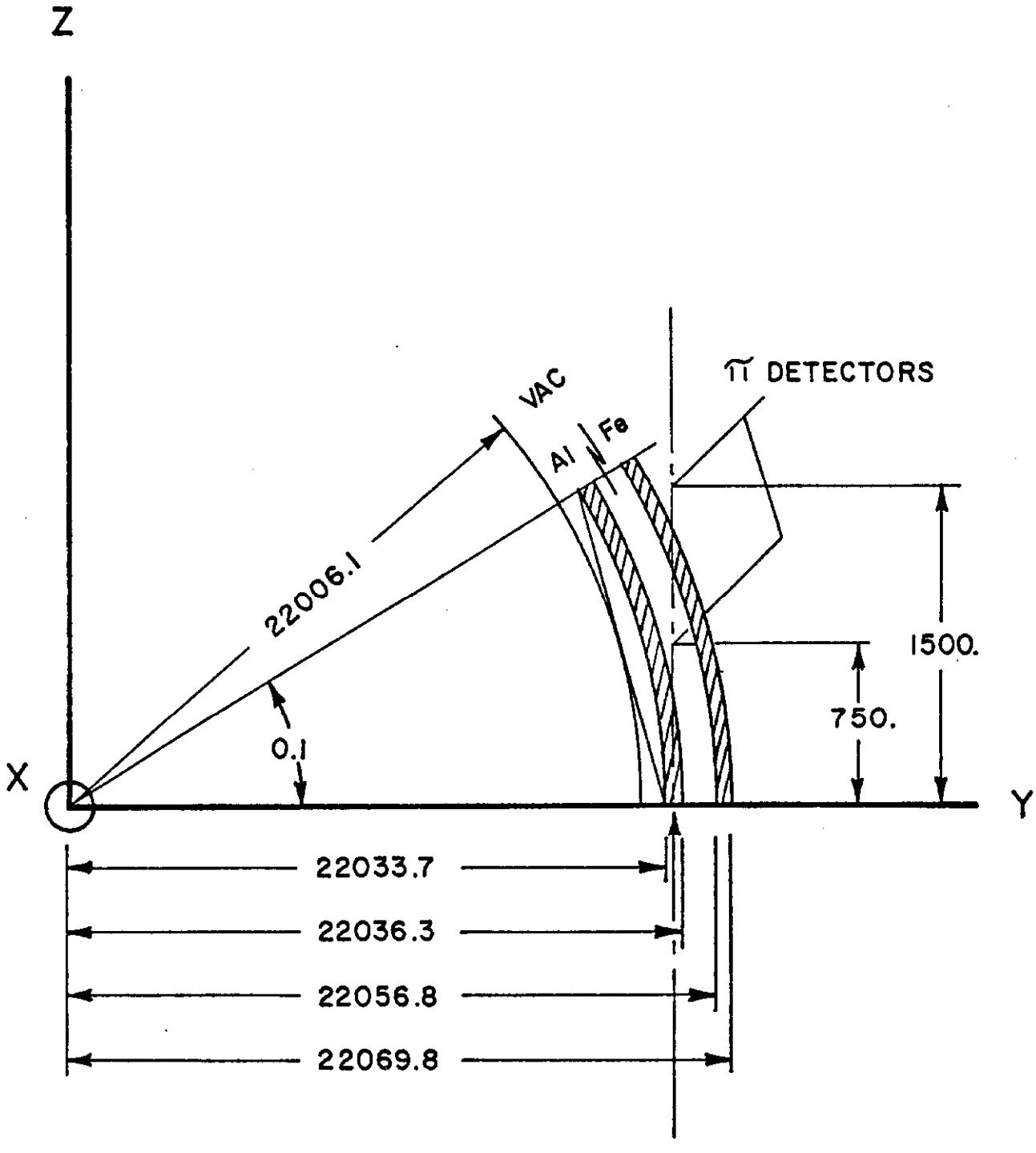

Z
VAC
Al
Fe
π DETECTORS
22006.1
0.1
1500.
750.
X
Y
22033.7
22036.3
22056.8
22069.8

Fig. 4